PROJECTILE WITH AIR RESISTANCE

V.B. Bhatia
Professor of Physics (Retired)
215/21A, Faridabad

It is interesting to know what effect the resistance of air has on the trajectory of a projectile. Here, we compute the trajectories for various amounts of air resistance.

We all know that, in the absence of any air resistance, the path of a projectile is a parabola. What happens to the path if air resistance is taken into account? Does it depart substantially from parabolic curve? Let us explore these questions.

For an object moving fast in air, such as a cricket ball bowled by a fast bowler (~144 km/h, or, ~ 40 m/s), the resistance due to air, also called air drag, can be taken to be proportional to the square of the velocity. If drag coefficient is denoted by *D*, and the velocity with which the projectile is launched is $\vec{v}$, then the magnitude of the drag force is

$$F = Dv^2. \tag{1}$$

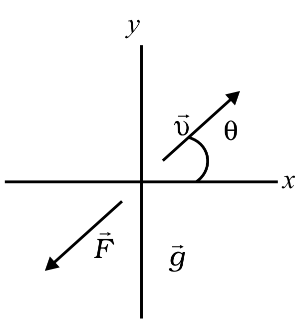

Fig. 1 : Showing all the relevant quantities

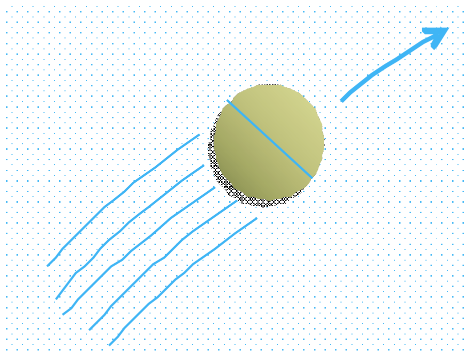

Fig. 2: A ball moving in air

The drag coefficient *D* depends on the area of the projectile facing the air in the direction of its motion and the density of air through which it moves. If *A* is the surface area of the projectile and ñ is the density of air, then drag coefficient can be written as:

$$D = C\frac{A\rho}{2}, \tag{2}$$

where the constant C takes care of the shape and nature of the surface of the projectile and other factors are not included here. The value of C may vary between 0.1 and 1.0.

The direction of the resistance or the drag force is always opposite to that of the velocity $\vec{\upsilon}$ (Fig. 1).

While the components of the velocity vector are

$$v_x = v\cos\theta,\ \ v_y = v\sin\theta, \qquad (3)$$

the components of the drag force acting on the projectile are

$$F_x = -Dvv_x,\ \ F_y = -Dvv_y\ . \qquad (4)$$

Consequently, the acceleration experienced by the projectile along the *x*– and *y*– direction is

$$a_x = -\frac{Dvv_x}{m}, \qquad (5)$$

$$a_y = -g - \frac{Dvv_y}{m} \qquad (6)$$

where g is the magnitude of the acceleration due to gravity and *m* is the mass of the projectile. Notice that, in accordance with the usual convention, all quantities measured upwards are taken as positive and those in the downward direction are taken as negative. Notice also, that the appearance of *D* and *m* in the expression for the acceleration will make the trajectory of the projectile depend on the mass and radius of the projectile. Recall that in the absence of any resistance these two factors do not come into our calculation of the trajectory.

Having got the expressions for the velocity and acceleration components, it is easy to write an algorithm to compute the trajectory. The algorithm can be converted into a computer programme, too.

We specify the initial position (*x*, *y*), mass and radius of the projectile along with the constant *C*. Specifying the initial velocity with which the projectile is launched and the angle at which it is launched, v_x and v_y can be found. These lead to the knowledge of a_x and a_y (Equations 5 and 6). We choose a suitably short interval of time during which we can assume the acceleration to remain constant. At the end of this interval

$$v_x \to v_x + a_x\Delta t \qquad (7)$$

$$v_y \to v_y + a_y\Delta t\ . \qquad (8)$$

The *x*-and *y*-coordinates advance to

$$x \to x + v_x\Delta t + \frac{1}{2}a_x(\Delta t)^2 \qquad (9)$$

$$y \to y + v_y\Delta t + \frac{1}{2}a_y(\Delta t)^2\ . \qquad (10)$$

The new velocity found in Equations 7 and 8 is fed back into Equations 5 and 6 to get the new acceleration. Steps 7, 8, 9 and 10 are then

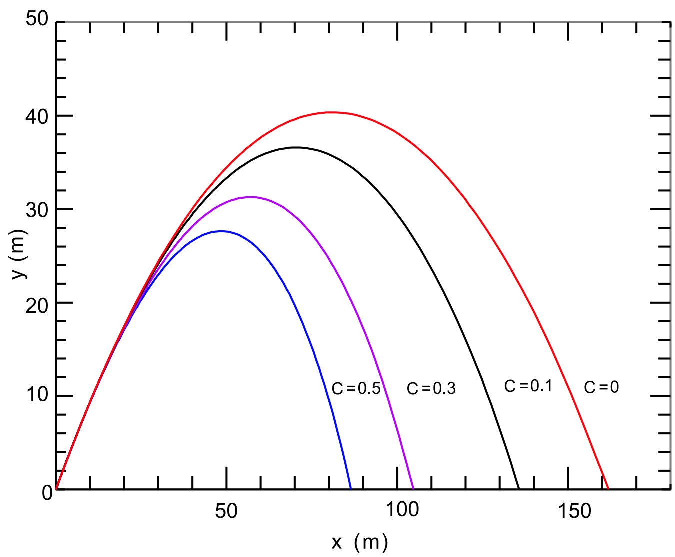

Fig. 3 : The trajectories of a projectile with varying resistance of air

repeated. The process is repeated a number of times and y is plotted against x to get the trajectory.

The trajectories in Fig. 3 have been plotted for a cricket ball travelling at a velocity of 40 m/s (144 km/h) at an angle of 45° to the horizontal. The data was generated by a computer programme. The mass of the cricket ball is taken as 0.160 kg and its radius as 0.0360 m (these are typical values). The value for the density of air has been adopted as 1.22 kg/m^3, appropriate for sea level at 15 °C. The values of the constant C are shown along each curve.

Conclusion: It is clear that the resistance due to air, at least in the case of high velocity projectiles, cannot be ignored while computing their trajectories. Not only does the projectile not reach its expected height, its range is also considerably reduced. Moreover the trajectory of the projectile becomes dependent on the mass and the shape of the projectile.

Here is a simple programme in power basic for computing x and y, which you can plot:

```
x = 0:y = 0:m = 0.160:rho = 1.22:r = 0.036:v = 40:g = 9.8: π = 3.14159:th = 45:t = 0  c = 0.5
d = rho*c* π *r*r*0.5:thd = π /180*th
vx = v*cos(thd):vy = v*sin(thd)
for i = 1 to 360                                  (start of loop)
 ax = – d/m*v*vx:ay = – g – d/m*v*vy              (updation of acceleration components)
 t = t + 0.0001                                   (updation of time, time step should be small)
 vx = vx + ax*t:vy = vy + ay*t                    (updation of velocity components)
 x = x + vx*t + 0.5*ax*t*t:y = y + vy*t + 0.5*ay*t*t   (updation of position coordinates)
 if (int(i/20) = (i/20) then print x,y            (prints every twentieth value)
 v = sqr(vx*vx + vy*vy)                           (updation of velocity magnitude)
next I                                            (end of loop)
```

By adding a few lines to the programme you can actually plot the curves, rather than noting x,y.