An Investigation into the Learning Difficulties Experienced by Indian School Children In Learning Division Algorithm of Simple Fractions

TAPAN KUMAR MAITI
Assistant Teacher
Hetia High School (H. S.)
JAYANTA KUMAR METE
Reader, Department of Education
University of Kalyani
Nadia, West Bengal 741 235

THE ONLY division algorithm for simple fractions which is being taught to the school children is the reciprocal algorithm. Apparently it is very simple. But, taught mechanically, it gives rise to many learning difficulties, which the present investigation intends to highlight for the benefit of students, teachers and method masters.

Introduction

After the introduction of cheap hand-held calculating machines, use of decimal fractions in daily life has become very convenient. The teaching-learning of decimal fractions has also undergone a sea change.

But matters have not been so much fortunate in case of public use or teaching-learning of simple fractions (also called "common" or "vulgar" fractions), though they have been as useful as before. There is no mechanical device to facilitate the use of such fractions. The teaching-learning of the algorithms of simple fractions has remained the same for decades.

Under these circumstances, the only ways to ameliorate the teaching-learning of simple fractions are to select the methods of operation for them most judiciously, and to follow up their teaching-learning with diagnostic programmes and remedial measures.

The Only Division Algorithm for Simple Fractions taught in Schools

Now there is one division algorithm for simple fractions that is used in the schools for teaching the school children. It is the reciprocal algorithm. Its rule is as follows:

To divide a number by a fraction multiply the number by the reciprocal of the fraction.

(The reciprocal of a fraction is the fraction formed by interchanging the numerator and denominator; for example, the reciprocal of $\frac{c}{d}$ is $\frac{d}{c}$).

Thus, in this method:

$$\frac{a}{b} \div \frac{c}{d} = \frac{a}{b} \times \frac{d}{c}$$

The rationale of the method is as follows.

$$\frac{a}{b}\,\frac{c}{d} \quad \frac{a}{b}\,\frac{d}{c} \quad \frac{c}{d}\,\frac{d}{c} \quad \frac{a}{b}\,\frac{d}{c} \quad 1 \quad \frac{a}{b}\,\frac{d}{c}.$$

So we see that, in this method, the divisor is reduced to unity by multiplying it by its reciprocal fraction. To keep

balance, the dividend is also multiplied by reciprocal of the divisor.

The major criticism of this method is that the teachers often forget to tell the students the mystery behind the inversion of the divisor, and then its use as the multiplier of the dividend to get the quotient.

Previous Researches on Simple Fractions

From a study of recent researches on simple fractions, it was found that the algorithms of simple fractions had not drawn the attention of investigators during the last 25 years. In 1930, Brueckner administered his 'Diagnostic Test in Fractions' to 400 sixth grade British pupils and made an analysis of the most frequent types of errors [Blair 1962, 231-234]. In 1979 Smith experimentally compared two methods (techniques) of division of fractions, namely, 'The common denominator method' and the 'Reciprocal Method' (also called the 'Method of Inversion') on their related gain in pupil achievement. The reciprocal was found by him superior to the common denominator method and is significantly increasing pupils' achievement in division of simple fractions [vide C. A. Smith, 'Effect of a Meaningful treatment for Division of Fractions a Comparative Study' (unpublished Doctoral Dissertation, the University of Texas at Austin, 1979). In India, in 1980 A. Bhattacharya conducted a doctoral study 'to diagnose and prevent the learning disabilities of Indian primary school students in simple and decimal fractions' [unpublished Doctoral Dissertation, University of Calcutta, 1980].

Need of Diagnostic Programmes

Faulty teaching-learning of the basic mathematical skills may lead to development of learning difficulties in a student. Uncorrected for long, the learning difficulties may develop into permanent-learning disabilities which may impede the learning of mathematics and career of the concerned student.

Hence diagnostic testing after the testing-learning of basic mathematical skills should be an integral part of a school programme. The data can help a sincere teacher to help the concerned students individually to ameliorate their learning. It would also help the teacher to formulate his/her teaching strategies so that the learning difficulties may not occur in his/her students in future

Need of the Study

Thus, the only work, investigators of present study, known to in the last 25 years that had considered Indian students' learning difficulties in simple fractions, was that of A. Bhattacharya (1980). It is unfortunate that the investigator paid little attention to division of simple fractions, probably thinking that it was only a modified version of fraction multiplication, which the 'Reciprocal' or 'Inversion' Method of division of fractions appears to be to many.

But experts like Brueckner [Ibid], Siddons [Godfrey and Siddons 1957, 123-124], Mueller [Mueller 1964,251-252], Adams, Ellis and Beeson [Adams, Ellis,

and Beeson 1977, 138] and many others have pointed out that, though apparently the reciprocal algorithm for division of fractions is very easy to use and practise, it is the major source of learning difficulties faced by the school students in doing division of fractions sums in arithmetic.

This is why the investigators took upon themselves the task of finding out, for the first time ever, the learning difficulties of the Indian pupils in solving division sums by the application of the reciprocal algorithm of simple fractions.

The Diagnostic Test used in the Study

"Brueckner Diagnostic Test in Division of Fractions" was published by the Educational Test Bureau, Minneapolis, Minnesota in 1930, 1943. The Test (vide Table) attempts to determine in a very thorough manner what mistakes pupils make and why they make them. It contains 40 types of division sums. A diagnostic tabulation sheet provided for it assists the teacher in interpreting the data and in outlining suitable remedial work. It had been used in the study to analyse students' difficulties in using the reciprocal method of division of fractions (vide Table).

Purpose of the Study

One of the investigators of the present study is a practising school teacher, the other a methods master. Both wanted to know the learning difficulties likely to be developed by Indian school students in the teaching-learning of the division algorithm for simple fractions. But they found no such recent study conducted on Indian school students. The knowledge was important for both of them, because the school teacher had to chalk out his teaching strategies in order to prevent development of learning difficulties amongst his pupils. And the methods teacher had to instruct the trainees in this matter.

Selection of the Experimental School

The investigators were in favour of a school in between low-achieving and high-achieving schools in which the students had been taught division of fractions in a rather casual and mechanical manner. Such a school usually contains all types of students. They found a school in the Bankura District which fulfilled all these requirements, and, besides, it was a large school, with near about 200 students in Class VI. It was selected as the experimental school, as its students were expected to show all learning disabilities arising out of faulty learning-teaching of division of fractions.

The other details about the experimental school are as follows:
Type of school: Higher Secondary with Arts and Commerce streams.
Number of students (average of last three years) : 1400 students.
Nature of location : Rural.
General academic qualifications of guardians: Very few are graduates or have higher qualifications.

The Sample

The sample for the study consisted of 101 students of Class VI selected randomly from 185 students of the class. They had

been taught the reciprocal method of division of fractions "by rote", by being made to use the method only mechanically, without having any explanation of the inversion of the divisor and then its use as the multiplier. Actually it was this ignorance of the students for which the investigators had selected their class, as they were likely to display all possible learning errors for Indian students who are victims of faulty teaching of the reciprocal method.

Of the 101 students, 45 were boys and 56 girls. Their average age was 12 years. As the IQs of the students were unavoidable, their average score in the last Half yearly examination of 281 marks and their average score in the last Arithmetic Test in the last Half yearly examination of 28 marks were obtained instead, as the correlation between scholastic aptitude and academic achievement is very high.

Administration of the Diagnostic Test

The test was administered on the subjects on 23.10.02 (Wednesday) in two sittings. The first sitting was held from 1.10 to 1.50 p.m., and the second, after the tiffin recess, from 2.20 to 2.55 p.m. One of the investigators supervised the testings.

Results

The following Table presents the obtained data grouped under possible categories of learning difficulties. The analysis in the Table is based entirely upon an examination of the pupils' answer sheets after they had been tested in a group. Consequently, the cause of some of the errors which were found could not be ascertained. This is shown in 2(c) and 10 of the Table under the caption "unknown". Nonetheless, the analysis would provide the teacher with a list of the major mistakes pupils make in dealing with problems on fractions. Individual diagnosis would reveal the unknown sources of error.

Findings of the Study

Following were the findings of the diagnostic study:

1. 31.70 per cent (that is, approximately one third) of all mistakes in division of fractions were due to the student's negligence to invert the divisor before performing the process of multiplication.
2. 24.70 per cent of all mistakes in division of fractions were due to the student's lack of comprehension of the process involved.

These support Mueller's observation that the "why" behind the reciprocal method is not widely or easily understood which accounts for the computational blunders [Mueller 1964, 251]. Here the misunderstanding reached the peak [(1) + (2) make up for 56.40 per cent of total errors made by the pupils] because the students had not been explained the logic behind the reciprocal method and were asked to use it mechanically without asking for the "why" of it. That is why the students inverted the dividend in place of the divisor (9.82 per cent of total errors were due to it), or inverted both dividend and divisor (13.82 per cent of total errors were due to it), so that "inverting" the wrong term accounted for

Table 1: Analysis of Difficulties in Division of Fractions

Different Types of Errors		Number of Errors	Per-centage
1. **Wrong operation: Multiplication:** $\left(1\frac{3}{8} \div 1\frac{2}{3}\right) = \frac{11}{8} \times \frac{5}{3} = \frac{55}{24} = 2\frac{7}{24}$		565	31.70
2. **Computation Errors**			
(a) Division: $3\frac{3}{8} \div 1\frac{3}{4} = \frac{27}{8} \times \frac{4}{7} = \frac{27}{14} = \frac{13}{14}$	3		
(b) Multiplication: $1\frac{1}{8} \div 3\frac{1}{2} = \frac{6}{5} \times \frac{2}{7} = \frac{12}{30} = \text{or } \frac{2}{5}$	54		
(c) Unknown: $3\frac{1}{3} \div 1\frac{3}{4} = \frac{10}{3} \times \frac{4}{7} = \frac{40}{21} = \text{or } 3\frac{1}{21}$	20		
		77	4.32
3. **Lack of Comprehension of process involved**			
(a) Inverts dividend: $1\frac{1}{5} \div 3\frac{1}{2} = \frac{5}{6} \times \frac{7}{2} = \frac{35}{12} = \text{or } 2\frac{11}{12}$	175		
(b) Inverts both dividend and divisor: $1\frac{3}{8} \div 1\frac{2}{3} = \frac{8}{11} \times \frac{3}{5} = \frac{24}{55}$	246		
(c) Adds denominators and multiplies numerators: $1\frac{3}{8} \div 1\frac{2}{3} = \frac{11}{8} \times \frac{3}{5} = \frac{33}{13} \text{ or } 2\frac{7}{13}$	1		
(d) Adds numerators and multiplies denominators: $1\frac{1}{5} \div 3\frac{1}{2} = \frac{6}{5} \times \frac{2}{7} = \frac{8}{35}$	2		
(e) Disregards denominator in quotient: $3\frac{1}{8} \div 1\frac{1}{4} = \frac{25}{8} \times \frac{4}{5} = 5$	5		
(f) Disregards numerator: $\frac{1}{9} \div \frac{1}{3} = \frac{1}{9} \times \frac{3}{1} = 3$	11		
		440	24.70

4. **Difficulty in Reducing Fractions to the lowest terms** (a) Does not reduce: $1\frac{1}{3} \div 3\frac{1}{3} = \frac{4}{3} \times \frac{3}{10} = \frac{4}{10}$ (b) Divides denominator by numerator: $1\frac{3}{8} \div 1\frac{2}{3} = \frac{11}{8} \times \frac{3}{5} = \frac{33}{40} = 1\frac{7}{33}$	57 –	57	3.20
5. **Difficulty in changing mixed numbers to improper fractions:** $3\frac{1}{3} \div 1\frac{3}{4} = \frac{10}{3} \times \frac{4}{12} = \frac{20}{18} = 1\frac{1}{9}$		84	4.71
6. **Omitted**		10	0.57
7. **Failure to reduce improper fractions to mixed numbers:** $3\frac{1}{8} \div 1\frac{1}{4} = \frac{25}{8} \times \frac{4}{5} = \frac{5}{2}$		201	11.27
8. **Errors in copying:** $1\frac{1}{4} \div \frac{1}{3} = \frac{5}{2} \times \frac{3}{1} = \frac{15}{2}$ or $7\frac{1}{2}$		174	9.76
9. **Cancellation Difficulties:** (a) Cancels within the denominators: $\frac{5}{6} \div 4 = \frac{5}{6} \times \frac{1}{4} = \frac{5}{6}$ (b) Cancels within the numerators: $1\frac{1}{5} \div 3\frac{1}{2} = \frac{6}{5} \times \frac{3}{7} = \frac{3}{35}$	3 9		
10. **Unknown:** $1\frac{1}{3} \div 3\frac{1}{3} = \frac{1}{5}$		12	0.67
		162	9.10
		1782	100.00

Notes: $\frac{565}{1782} \times 100 \times 31.70$

23.64 per cent of total errors committed. As stated, non-inversion accounted for 24.70 per cent errors. The similarity of the findings of the two diagnostic testings held 75 years apart upon children of two different nations cannot escape our notice.

One of the testings was Brueckner's held in 1930 upon 400 British school children. The other was by the investigators' held in 2002 upon 101 Indian school children of the same grade and age group. In both the groups of children, nearly one-third missed to invert the divisor before using it as the multiplier of the dividend. But there were also dissimilarities in findings. Whereas 13.8 per cent of the British pupils committed computation errors, it was only 4.32 per cent in case of the Indian pupils, on the other hand, while 12.1 per cent of the British pupils committed errors for "Lack of Comprehension of the process involved", the figure was double (24.70 per cent) for the Indian pupils.

Suggestions for Prevention of Learning Difficulties

Godfrey and Siddons think that the rule of the reciprocal method of division of fractions, viz., "Turn the divisor upside down and multiply by that" is quite unnecessary and certainly dangerous for the beginners. Instead of beginning with the method, Siddons has suggested the following method to start with [Godfrey and Siddons 1957, 123-124]. "If the children are clear that $5 \div 8 = \frac{5}{8}$, it is natural to assume that $\frac{3}{4} \div \frac{2}{5} = \frac{3/4}{2/5}$

"This fraction has four "stories" and we only want two. Obviously we should improve matters by multiplying top and bottom by 4 and also 5.

$$\therefore \frac{3}{4} \div \frac{2}{5} = \frac{3/4}{2/5} = \frac{3/4 \times 4 \times 5}{2/5 \times 4 \times 5} = \frac{3 \times 5}{2 \times 4} = \frac{15}{8} = 1\frac{7}{8}.$$

Siddons expects that (see above) many pupils will discover the "reciprocal method" herefrom.

Siddons has also pointed out two other benefits (other than preventing the development of learning difficulties in the division of fractions) of the treatment suggested by him [Ibid, 124] :

1. "In the treatment I have suggested; the whole argument is made to hang on the one rule, 'the golden rule about fractions' —

   'The value of a fraction is unaltered by multiplying (or dividing) numerator and denominator by the same number.'
2. Exactly the same principles are used when dealing with fractions in algebra."

The investigators however are of the opinion that, as the reciprocal method is the easiest division algorithm of fractions, it will be desirable if it is taught and learned with proper understanding of its logic. So the beginner will work out the whole process as the following worked out problem, and will themselves abbreviate the method.

Example 1 $\frac{2}{3} \div \frac{4}{15}$. The reciprocal of $\frac{4}{15}$ is $\frac{15}{4}$. Multiplied by it, the divisor will

be reduced to unity. But to keep balance, the dividend will also be multiplied by $\frac{15}{4}$.

$$\therefore \frac{2}{3} \div \frac{4}{15} = \left(\frac{2}{3} \times \frac{15}{4}\right) \div \left(\frac{4}{15} \times \frac{15}{4}\right) = \left(\frac{2}{3} \times \frac{15}{4}\right) \div 1 = \frac{2}{3} \times \frac{15}{4}.$$

Example 2

$$1\frac{3}{5} \div 3\frac{1}{5} = \frac{8}{5} \div \frac{16}{5} = \left(\frac{8}{5} \times \frac{5}{16}\right) \div \left(\frac{16}{5} \times \frac{5}{16}\right) = \left(\frac{8}{5} \times \frac{5}{16}\right) \div 1 = \frac{8}{5} \times \frac{5}{16}.$$

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