Two Typical Projectile Problems and their Solution

S.N. MAITRA
Department of Mathematics
National Defence Academy
Khadakwasla, Pune

IN THIS ARTICLE two problems on projectile motion have been set forth and solved with a view to help students and teachers involved in learning teaching of mathematics at higher secondary stage. Such types of problems are also relevant to physics students for entrance tests in professional courses.

Problem 1

A shell is fired from ground level so as to hit an enemy position, after passing over a hill, situated at the foot on its other side at same level. The shell passes just over the peak of the hill and strikes the target. The horizontal distances of the peak and the target are *a* and *b* respectively. If *h* be its height and *g* the acceleration due to gravity (Figure. I), find the velocity and angle of projection of the shot. Also find how long the shell is over the hill after grazing its peak. Neglect the air resistance.

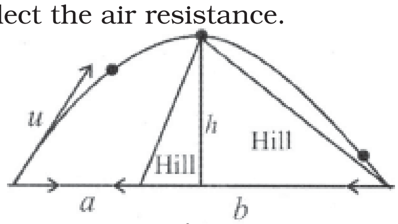

Fig. 1

Solution

Let the shell be fired with velocity *u* at an angle   to the horizontal and *t* be the time taken by it to reach the peak of the hill.

Considering its vertical and horizontal motion,

$$h \quad (u \sin \quad) t \quad \frac{1}{2} g t^2 \tag{1}$$

$$a \quad u(\cos \quad) t \tag{2}$$

Eliminating *t* between Eqs. (l) and (2), one gets

$$h \quad a \tan \qquad \frac{1}{2} g \frac{a^2}{u^2 \cos^2} \tag{3}$$

Since to hit the enemy position at the foot of the hill at the same ground level, the horizontal range of the shell is:

$$b \quad \frac{u^2 \sin 2}{g} \tag{4}$$

Eliminating u^2 between Eqs. (3) and

$$h \quad a \tan \qquad \frac{a(b \quad a)}{b} \tan$$

$$or \quad \tan \qquad \frac{hb}{a(b \quad a)} \tag{5}$$

Using Eq. (5) in Eq. (4) we have the velocity of projection

$$u^2 \quad \frac{bg}{\sin 2} \quad \frac{bg\,(1 \quad \tan^2 \quad)}{2 \tan}$$

$$\frac{g \;\; a^2\,(b \quad a)^2 \quad h^2 b^2}{2h\,(b \quad a)\,a} \quad (\text{for } b \quad a) \tag{6}$$

If b = $2a$, h, that is, the hight of hill top becomes equal to the greatest height attained by the shell. Then this problem reduced to a textbook problem[1]

Combining Eqs. (5) and (4), we get

$$u^2 \quad g \; \frac{b^2}{8h} \quad 2h$$

$$u \; sin \qquad \sqrt{\frac{bg}{2} \tan a} \qquad \sqrt{\frac{b^2 \; gh}{2a\,(b \quad a)}} \qquad (7)$$

The time of travel T by the shot over the hill between the peak and the target is given by using Eqs.(1) and (7) T = total time of flight – time to rise to the peak

$$\frac{2u \sin}{g} \quad t$$

$$\frac{2u \sin}{g} \quad \frac{1}{g}(u \sin \qquad \sqrt{u^2 \sin^2 \qquad 2gh})$$

$$\sqrt{\frac{hb^2}{2ga\,(b \quad a)}} \mp \sqrt{\frac{hb^2}{2ga\,(b \quad a)} \quad \frac{2h}{g}}$$

$$\sqrt{\frac{h}{2ga\,(b \quad a)}} \quad b \mp \sqrt{b^2 \quad 4a\,(b \quad a)}$$

$$\sqrt{\frac{2ha}{g\,(b \quad a)}} \quad or \quad \sqrt{\frac{2h\,(h \quad a)}{ga}}$$

according as and $\frac{b}{2} \quad a \quad b$ and $a \quad \frac{b}{2}$

If the $b = 2a$, T = $\frac{\sqrt{2h}}{g}$ as is in textbook[1].

Problem 2

A particle is projected from the foot of an inclined plane of length L and inclination to the horizontal. It grazes the top of the inclined plane at a subsequent time T and thereafter reaches the same horizontal level at the foot of the inclined plane. Find the velocity, angle of the inclined plane and its time of travel over the horizontal plane. Assume acceleration g due to gravity to be a constant and neglect the air resistance. (Figure 2). What can be maximum distance of the projectile from the inclined plane.

Solution

Let the particle be projected with a velocity u at an angle to the horizontal.

Considering its horizontal and vertical motion during time T of flight over the inclined plane.

$$L cos \quad = u\,(\cos \quad)\,T \qquad (1)$$

$$L \sin \quad = u\,(\sin \quad)\;T \quad \frac{1}{2} gT^2 \qquad (2)$$

Eliminating L between Eqs. (1) and (2) or by formula for flight over the inclined plane we have

$$T \quad \frac{2u \sin (\qquad)}{(g \cos \quad)} \qquad (3)$$

Eliminating u between Eqs. (1) and (3)

$$\frac{\sin (\qquad)}{\cos} \quad \frac{gT^2}{2L}$$

$$\text{or} \quad \tan \qquad \tan \qquad \frac{gT^2}{2\,L \cos} \qquad (4)$$

In consequence of Eqs. (4), Eq. (1) gives the velocity of projection

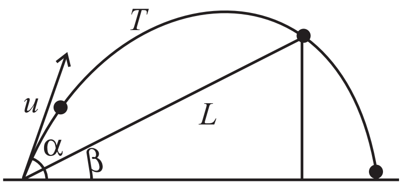

Fig. 2

$$u \quad \frac{L \cos}{T} \sec \qquad \frac{L \cos}{T} \sqrt{1 \quad \tan^2}$$

Substituting value of tan a from Eq. (4)

$$\text{or } u \quad \frac{L}{T} \quad 1 \quad \frac{gT^2 \sin}{L} \qquad \frac{g^2T^4}{4L^2} \quad ^{\frac{1}{2}} \qquad (5)$$

If t be the time of its flight over the horizontal plane,

t = total time of flight – time of flight over the inclined plane

$$\frac{2u \sin}{g} \qquad \frac{2u \sin(\quad)}{g \cos} \qquad \frac{2u \cos \quad \sin}{g \cos}$$

Using Eq. (1), we get

$$t \quad \frac{2L \sin}{gT} \qquad (6)$$

The distance of the projectile from the inclined plane at time t is

$$y \quad u \sin(\quad) t \quad \frac{1}{2} gt^2 \cos \qquad (7)$$

For maximum if $\frac{dy}{dt} = 0$ from Eg. (7) gives because of Eg. (3)

$$t \quad t_{opt} \quad \frac{u \sin(\quad)}{g \cos} \quad \frac{T}{2} \qquad (8)$$

$$y_{\max} \quad \frac{1}{8} g (\cos \quad) T^2$$

REFERENCE

M. RAY and G.C. SHARMA. 1990. *Textbook on Dynamics*, S. Chand & Co. Ltd. New Delhi, pp-88-102.