A COMPARITIVE STUDY OF SCHOLASTIC ACHIEVEMENT IN MATHEMATICS EXAMINATION IN RELATION TO CONCEPTUAL UNDERSTANDING AND MATHEMATICS ABILITY

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The study compares two groups of students of Class VII and Class IX, one group belonging to government schools and the other to private schools. Scholastic achievement in examinations was studied in relation to conceptual understanding and mathematics ability. Conceptual understanding and mathematical ability both exhibited positive relationship with scholastic achievements. It was also seen that conceptual understanding and mathematics ability of the students was at a lower level than scholastic achievement. Some common conceptual errors in mathematics were also discussed.

Introduction

The culture of mathematics education highly values student performance and meaningful learning by students (Gold, A. G. 2003). Problems in mathematics education are universal and not new. They have existed from primary to higher levels of education. Many students cannot understand what was taught in mathematics classrooms because the emphasis is on memorising procedures without any meaningful context.

The process of teaching mathematics in schools include procedural skills as well as conceptual knowledge. Assessments provide a systematic way to inform students, teachers, parents and educationists about student performance. Hence, the need for examinations . However, at the same time, the motivation, commitment and the imaginative faculties of the learner in any discipline cannot be evaluated through a one-shot examination at the end of an academic year. Mathematics is now presented as an interdisciplinary subject. Mathematics knowledge becomes meaningful and powerful in application. This is possible only when there is a thorough understanding of basic concepts.

Based on an understanding of the basic concepts the students construct and develop new ideas and skills, and the processes they learn become richer and more complex. Students then have a better understanding of how topics fit together as well as develop a greater confidence in problem-solving .

There is a long history of research, going back to the 1940s and the work of William Brownell (1945), on the effects of teaching for meaning and understanding in mathematics. Mathematical concepts have to be conveyed to the students by their teachers so that they can apply and retain information through deep understanding. Although students must develop basic mathematical skills, the processes, concepts and understanding should take precedence.

Method

Objectives

The main objectives of this study were:

- to compare the mathematics achievement in examinations, knowledge of basic concepts from syllabus and ability for mathematics of secondary and high school students of undivided Bongaigaon district.
- to compare the above results with respect to government and private schools.
- to determine the relationship among the above three variables.
- to find out some common conceptual errors.

Hypotheses

Keeping in view the objectives stated above the following null hypotheses were proposed for the present study

- H1: There is no significant difference in the mean of variable scholastic achievement and the mean of the variable conceptual understanding for Class IX.
- H2: There is no significant difference in the mean of variable scholastic achievement and the mean of the variable mathematical ability for Class IX.
- H3: There is no significant difference in the mean of variable scholastic achievement and the mean of the variable conceptual understanding for Class VII.
- H4: There is no significant difference in the mean of variable scholastic achievement and the mean of the variable mathematical ability for Class VII.
- H5: There is a significant relationship among the three variables scholastic achievement , conceptual understanding and mathematical ability for Classes IX and VII.

Sample

Bongaigaon district of Assam was the field area for the study. The field area has been divided into Government Schools and Private Schools. In order to cover the different strata of population, a stratified random sampling of schools have been taken .The sampling size was taken as 30. By the proportional allocation method, the number of government schools taken were 24 and the number of private schools taken were 6.The schools were selected with the help of the Fischer Yate's table. For the purposes of this study Classes VII and IX were chosen. A simple random sampling of ten students each from Classes VII and IX have been taken from each school by the lottery method. The total number of samples in Class VII and Class IX being 290 each.

Tools

For the purpose of this study, the following tools were used.

Test for knowledge of mathematical concepts

These were teacher-prepared tests. These tests were developed from contents within school syllabus common to both SEBA and CBSE schools. All items in this test were selected on basis of the contents included in the standard mathematics textbooks used by the schools which were surveyed.

Test of mathematical ability

These were teacher-prepared tests designed to emphasise mathematical ability over achievement.

This test was designed to place less emphasis on computational skills and more emphasis on students' visual/spatial skills, pattern recognition, and logical reasoning skills. Instead of questions of the textbook type, the question paper had problems and puzzles which would test the understanding of the students in the areas of reasoning and visualisation.

Test of mathematics achievement

The criteria for measuring the mathematics achievement in examination was taken as the mathematics marks which a student received at the end of the academic year. This was expressed as a percentage with weightage given to unit tests, half yearly examination and annual examination . The marks were collected from the school records of the schools visited.

Statistical Analysis

The data was entered into a SPSS spreadsheet and was analysed accordingly. The mean and standard deviation of the combined scores were calculated. Different cross tabulations were constructed to show the joint distribution of two or more variables. t-test was used to examine the null hypotheses formulated on the basis of objectives. Regression analysis was used to investigate the relationship among the variables.

In order to test the null hypotheses, there exists no significance difference between scholastic achievement , conceptual understanding, and mathematical ability. The mean scores on different tests were calculated and the t-test was applied to find if the differences were statistically significant.

Cross table distribution of mean scores of the tests reveal that a student having both the attributes of very high achievement and very high conceptual knowledge forms only 0.3 per cent from Class IX and 0 per cent from Class VII, while for mathematics ability 0 per cent from Class IX and 0.3 per cent from Class VII.

The results shown in Table 1 reveal that conceptual knowledge and mathematics ability of the students were lower than their scholastic achievement. In all three areas of scholastic achievement, conceptual knowledge and mathematics ability, the pupils from the government schools came behind those of the privately-managed schools.

Table 1: Comparison of means among the different tests

	CLASS VII		CLASS IX	
MEAN SCORES	Government Schools	Private Schools	Government Schools	Private Schools
Scholastic achievement	47.94	68.12	46.03	75.25
Conceptual knowledge	20.73	34.24	20.74	40.77
Mathematics ability	13.24	30.20	16.12	36.67

All t values as seen in Table 2 are greater than the critical value, so the null hypotheses are rejected. This shows that there is significant difference between the means of the above pairs of variables.

Table 2: Paired samples test

	Mean	S.D.	t	Sig
Scholastic achievement Conceptual knowledge (IX)	27.18966	13.3747	34.619	**
Scholastic achievement Mathematical ability (IX)	31.70690	14.7007	36.729	**
Scholastic achievement Conceptual knowledge (VII)	28.59207	12.4244	39.189	**
Scholastic achievement Mathematical ability (VII)	35.37310	12.7454	47.262	**

P←.001

To study the effect of the independent variables on the dependent variable regression analysis has been employed. Here, scholastic achievement for Class IX has been taken as the dependent variable Y1 while conceptual understanding X1 and mathematical ability X2 have been taken as the independent variables. Similarly, scholastic achievement for Class VII has been taken as the dependent variable Y2 while conceptual understanding X3 and mathematical ability X4 have been taken as the independent variables.

The beta coefficients as seen in Table 3 are the estimates which refer to the expected change in the dependent variable, per standard deviation increase in the predictor variable. The positive value of the coefficients for conceptual understanding and mathematical ability indicate positive relationships with scholastic achievements. An increase in the value of each of these predictors will show an increase in the combined scores of the students. The statistics signifies that ß and ß1 are significant at P ← .01 per cent . However ß2 ,the regression coefficient for mathematics ability is significant at P ← .1 per cent .

Table 3: Model summary: Class IX

Co-efficient	Value	Std Error	t	Sig
ß	19.516	1.464	13.329**	.000
ß1	.808	.131	9.535**	.000
ß2	.044	.130	.517	.605

R2 = .721 F = 371.25

F = 371.25 shows that the means of the three variables are significantly different. R2 is the statistical measure of how well a regression line approximates real data points. Here, R2 = 0.721 shows that the regression explains 72 per cent of the variation on Y1 due to X1 and X2 . Thus, knowledge of basic concepts and mathematics ability of the students has a causal effect on the mathematics achievement of the students of Class IX and hypothesis V is accepted.

The regression equation thus is given by Y1 = 19.516 + 0.808 X1 + 0.044 X2, i.e., scholastic achievement =19.516 +.808 knowledge of basic concepts + 0.044 mathematics ability.

Table 4 gives the model summary for Class VII. The positive value of the coefficients for conceptual understanding and mathematical ability indicate positive relationships with scholastic achievements. An increase in the value of each of these predictors will show an increase in the combined scores of the students.

t statistics signifies that ß and ß1 are significant at P ←.01 per cent . However ß2, the regression coefficient for mathematics ability is significant at P←.1 per cent . F =200.475 shows that the means of the three variables are significantly different. R2 = 0.583 shows that the regression explains 58 per cent of the variation on Y2 due to X3 and X4. Thus knowledge of basic concepts and mathematics ability of the students has a causal effect on the mathematics achievement of the students of Class VII and hypothesis V is accepted.

Table 4: Model summary : Class VII

Coefficient	Value	Std. Error	t	Sig
ß	27.789	2.153	12.909**	.000
ß1	0.568	0.181	4.714**	.000
ß2	0.203	0.151	1.681	0.094

R^2 = .583 F = 200.475

The regression equation thus is given by Y2 = 27.789 +. 568 X3 + .203 X4, i.e., scholastic achievement =27.789 +. 568 knowledge of basic concepts +.203 mathematics ability.

Analysis of some common conceptual errors

The students' answers were marked 0 for no response, fractional marks for inadequate responses which contained major computation errors, responses which were attempted partially and responses which focussed entirely on the wrong mathematical idea or procedure and full marks for adequate responses which were clear and unambiguous, communicated effectively and showed mathematical understanding of the problem's ideas and requirements.

CLASS VII

Answer the following questions:

1. Evaluate (y + x) x when x = 5 and y = 6

Table 5: Student answers to Question 1

Marks (Max. 2)	Government Schools	Private Schools
Not attempted	67	5
0	30	2
0.5	26	6
1	42	13
1.5	25	18
2	30	16
Total	230	60

Students were unable to replace the literals x and y with their numerical values. There were cases where even if the numbers were replaced inside the bracket, the literal x in the exponent was not replaced. Further, there was no proper expansion of the exponent power.

2. Find the values of x, y , z

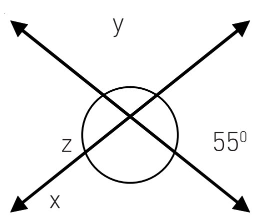

Table 6: Student answers to Question 2

Marks (Max. 3)	Government Schools	Private Schools
Not attempted	23	11
0	28	2
0.5	21	10
1	–	6
1.5	–	8
2	58	23
Total	230	60

A common error being all angles were shown as 55°. In certain cases, the value of x was found but not the other angles. Thus, the concept of the measure of a straight line being 180 was not known as also the proposition about vertically opposite angles being equal.

3. Find the area of the following figure

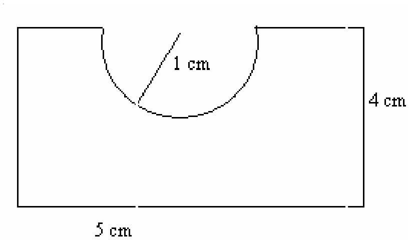

Table 7: Student answers to Question 3

Marks (Max. 3)	Government Schools	Private Schools
Not attempted	110	16
0	21	4
0.5	5	–
1	38	20
1.5	12	6
2	10	4
2.5	24	3
3	10	7
Total	230	60

There was glaring ignorance of the formula for areas of rectangle and circle. In some cases, the wrong formula, i.e., for perimeter of the figures were applied. In the cases, where the students showed the knowledge of the area of the rectangle and semicircle separately, they were unable to apply the concept of difference of two areas for a combined figure .

4. If AB, AC, AD and AE are parallel to line l, what can be said about the points A ,B ,C, D, E.

Table 8: Student answers to Question 4

Marks (Max. 3)	Government Schools	Private Schools
Not attempted	193	47
0	21	9
0.5	16	3
1	0	0
1.5	0	0
2	0	0
2.5	0	0
3	0	1
Total	230	60

In the government schools, 83.91 per cent and 77.96 per cent students of the private schools did not attempt the question showing an immense lack of visualisation skills. Common mistakes were the lines AB, AC, AD and AE being taken as different lines. In some cases, the answer was concurrent points.

5. Three clocks strike after 15, 20 and 30 minutes interval, respectively. If they all ring together at 6pm, when is the next time that they will ring together?

Table 9: Student answers to Question 5

Marks (Max. 3)	Government Schools	Private Schools
Not attempted	113	14
0	55	23
0.5	0	3
1	20	5
1.5	0	–
2	21	2
2.5	0	2
3	21	10
Total	230	59

The application of the concept of LCM was not known to the majority. In some cases, the computation of the LCM was done wrongly. In some cases, the HCF was computed instead of LCM. Common errors included the answer being 12'O clock. In some cases, the sum total of the minutes were added and then added to 6pm making it 7.05 pm.

CLASS IX

Answer the following questions:

1. Factorise $x^4 + y^4$ triangles.

Table 10: Student answers to Question 1

Marks (Max. 3)	Government Schools	Private Schools
Not attempted	30	6
0	172	22
0.5	10	2
1	0	3
1.5	0	–
2	4	2
2.5	14	5
3	0	20
Total	**230**	**60**

2. Divide 4x3 -12x2 +14x -3 by 2x -1.

Table 11: Student answers to Question 2

Marks (Max. 3)	Government Schools	Private Schools
Not attempted	23	5
0	72	4
0.5	17	6
1	118	36
1.5	0	1
2	0	10
Total	**230**	**60**

Errors were seen in the computation of basic operations on algebraic terms. The common error was that the process of division was stopped even though the degree of the remainder remained the same as that of the quotient, because there was the inability to use a fractional remainder in the quotient

3. In an isosceles triangle prove that altitude from vertex bisects the base.

Table 12: Student answers to Question 3

Marks (Max. 3)	Government Schools	Private Schools
Not attempted	36	26
0	10	8
0.5	2	1
1	2	3
1.5	2	1
2	–	2
2.5	2	3
3	1	16
Total	**230**	**60**

The application of the congruence criteria of triangles for solving the problem showed a lack of comprehension of the congruence criteria for only two instead of three identical parts of the triangles could be identified. In some cases, where three parts were identified they were not conformable with the congruence criteria , e.g. the SSA was taken as a criteria.

4. The length and breadth of a rectangular park are in the ratio 8:5. A path 1.5 metres wide running all around the outside of the park has an area of 594m. Find the length and breadth of the park.

Table 13: Student answers to Question 4

Marks (Max. 3)	Government Schools	Private Schools
Not Attempted	187	36
0	623	1
0.5	10	11
1	10	5
1.5	0	2

2	0	2
2.5	2	1
3	0	2
Total	**230**	**60**

Common errors were ignorance of the formula for finding the area of rectangle, using the wrong formula (the perimeter formula), inability to form a simple linear equation, inability to solve the equation. Thus, it was seen that the students lacked the consolidating ability which required combining the different areas of mathematics into a single problem.

5. An electric fan is listed at Rs 350 and is sold at a discount of 8 per cent. During off season, the shopkeeper announces a further discount of 8 per cent. Find the selling price of the fan and how much he would lose if he had announced a single discount of 16 per cent instead of two successive discounts of 8 per cent.

Table 14: Student answers to Question 5

Marks (Max. 3)	Government Schools	Private Schools
Not attempted	104	27
0	5	0
0.5	11	3
1	46	5
1.5	23	7
2	23	2
2.5	4	5
3	14	11
Total	**230**	**60**

There were errors in computation of percentage; instead of successive discount, single discount was taken in both cases. In cases where successive discount was taken, the second percentage for successive discount was calculated on original cost price instead of reduced cost price, operation of addition instead of difference was used to calculate the discount.

Discussion and Conclusion

The above analysis shows that the students failed to develop an understanding of the underlying mathematics concept of a problem. The percentage of all correct responses for the questions was much less than 50 per cent in all the cases. High achievement in mathematics based on examination marks may not reflect knowledge of basic concepts or the mathematics ability of the student. Among the students of this study, it is seen that pupils with high scholastic achievement were able to respond with success to examination questions but were not able to solve conceptual questions with accuracy and understanding. There are a number of common conceptual errors as shown . Not only should students have a fluency in basic computational skills but they must also develop an understanding of mathematical concepts. There is a minimal depth of conceptual understanding, they need in order to continue deepening their understanding in a subsequent course.

This study reveals unequivocally that the ability to memorise facts does not necessarily imply understanding of a concept (Stamovlasis *et al.*, 2004, Mohd. Sahar Sauian, 2002; Parmjit 2002; Reys, R. E. and Yang, D. C. 1998, Yager, 1991). G. K. Mainka (1983) who studied the understanding and the acquisition of mathematical concepts of pupils found that the majority of pupils who were promoted to the next grade did not show acquisition of concepts of the lower grade. Thus, apart from procedural skills which include student's ability to demonstrate appropriate use of procedures, conceptual understanding which includes the students' ability to interpret the problem and select appropriate information to apply a strategy for solution is an area which should be developed in order to improve the quality of mathematics education. Educators should shift from memorisation of facts and algorithms towards instruction that involves students in mathematical concept construction (Mayer and Jones, 2004).

The findings in this research paper also highlight the differences in all three variables among government and private schools. This indicates the existence of other environmental and socio-economic factors which may affect the variables. The factors affecting these should be identified and the disadvantages removed in order that the students are not limited in their achievement of mathematics education. School environment is a vital factor for imparting quality education and equal opportunities for quality education must be available across all sections of the society

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