COMPLEXITIES ABOUT SIMPLE HARMONIC MOTION

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Study of Simple Harmonic Motion (SHM) is not as simple as it appears to be. It has a number of complexities. The present paper deals with the concepts held by experienced teachers pertaining to sign for displacement, velocity, and acceleration, its effect on speed during different time intervals and direction of acceleration while executing SHM Responses obtained in training programmes have been analysed and indicate several misconceptions. Treating the path of an oscillating pendulum along a straight line particulalry poses a lot of difficulty. In view of these concepts having direct implications on teaching to students, strategy for teaching the related concepts pertaining to SHM, in particular, and problem-solving, in general, using basic cocepts and principles has been emphasised.

Introduction

One of the instructional objectives in teaching Physics at Intermediate level is to facilitate the learners in developing problem-solving competencies. It affects the learners in a number of ways such as promoting knowledge development, critical thinking, searching logic, ability, seeing relationships, capability to analyse an unfamiliar situation and synthesis of different ideas in a given problem, divergent thinking, interpreting a concept, drawing fig.1 inferences and so on. All these competencies lead to construction of new knowledge instead of putting thrust on memorisation of facts, rules, principles, laws and derivation of formal. It is however, observed that many problem solvers, students and teachers both, jump to solve a problem by using a formula instead of starting from basic concepts and principles. A study in this connection pertaining to teach about simple pendulum was carried out for three years in training programmes. Total number of participants in the programmes were eighty. Each participant had an experience of teaching at + 2 level for not less than two years.

Teaching about simple pendulum at +2 level includes key points such as simple harmonic motions under limited conditions, measurement and calculation of time period with varying lengths of pendulum, discussion on acceleration proportional to displacement and oppositely dissected, equation of simple harmonic motion, potential and kinetic energy and its transformation during oscillation of the pendulum and so on. Keeping these teaching points in view, following two problems were taken-up in order to determine teachers concepts during the training programmes.

Problems

(I) For simple harmonic motion represented as in Fig. 1, write the sign for displacement, velocity and acceleration for different time intervals viz. (0 –T/4), (T/4 – T/2), (T/2 – 3 T/4) and (3 T/4 – T) and also mention if the speed is increasing or decreasing.

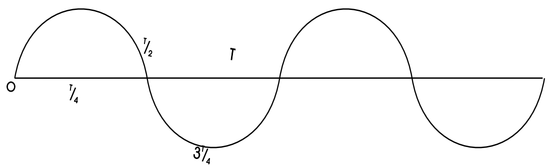

Fig. 1

(2) Draw the direction of acceleration for the care of a simple pendulum for the positions A, B, C, D and E (Fig. 2). Give reason for the same.

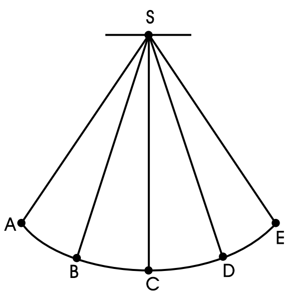

Fig. 2

Analysis of Responses (Problem 1)

Table1 gives the percentage of correct responses for question number (1), obtained from the participants along with the carried answer in the bracket:

These responses followed by discussion indicated that:

(i) Mostly the participants thought the solution to this problem taking into consideration an oscillating pendulum. For the time interval (0 – T/4), 97.5 per cent considered the displacement to be positive because of its being in the positive direction with respect to mean equilibrium position. However, only 67.5 per cent considered velocity to be positive and 68.7 per cent thought acceleration to be negative thinking that the speed of the pendulum is seen to be decreasing during this time interval.

(ii) Relatively smaller percentage of about 66.2 per cent considered displacement to be positive during the time interval (T/4 – T/2). Rest of the participants considered the displacement from the extreme position at T/4 and took it therefore, to be negative. Interestingly while 67.5 per cent correctly considered the speed to be increasing as per observations, only 38.8 per cent and 40 per cent considered the velocity and acceleration respectively to be negative. Others thought them to be positive because of increasing speed for this time interval.

Table 1: Responses of Participants

Time Interval	Displacement	Velocity	Acceleration	Speed
$0 - {}^{T}/_{4}$	97.5 (+)	63.7 (–)	68.7 (–)	67.5 (Decreasing)
${}^{T}/_{4} - {}^{T}/_{2}$	66.2 (+)	67.5 (+)	40 (–)	67.5 (Increasing)
${}^{T}/_{2} - 3\,{}^{T}/_{4}$	73.7 (–)	38.8 (–)	43.7 (+)	66.2 (Decreasing)
$3{}^{T}/_{2} - T$	71.2 (–)	50 (+)	75 (+)	66.2 (Increasing)

(iii) For the time interval (T/2 – 3T/2), 73.7 per cent participants correctly considered the displacement to be negative as it was in negative direction with repeat to mean equilibrium position. Also, 63.7 per cent correctly considered the velocity to be negative but 43.7 per cent rightly considered acceleration to be positive. Other participants wrongly thought the acceleration to be negative because of decreasing speed for this time interval.

(iv) For the time interval (3T/4 –T), 71.2 per cent correctly thought the displacement to be negative because of its being in negative direction with respect to mean equilibrium position. Other wrongly thought it from the extreme position at 3T/4. While only 50 per cent correctly thought velocity to be positive, a large percentage of about 75 per cent also correctly considered acceleration to be positive because of increasing speed towards the mean position from the position at 3T/4.

Conceptual Approach

These results indicate that while solving this problem, one may treat the pendulum to be oscillating in a straight line. Besides,

(a) Emphasis on considering reference point (in the present case the mean position) to be the same for all time intervals has to be made while determining the sign for displacement.

(b) For determining the sign for velocity, the direction of the change in displacements per unit time with respect to mean position together with their magnitude is to be considered. To do so, two displacement vectors, one corresponding to any instant in a given time interval and another corresponding to an instant preceding to it in the same time interval with respect to mean position can be drawn and their difference can be obtained.

(c) For determining the sign for acceleration, the direction for change in velocity per unit time together with its magnitude should be considered. To do so, two velocity vectors, one corresponding to any instant in a given time interval and another corresponding to an instant preceding to it in the same time interval, both with respect to mean position can be drawn and their difference can be obtained.

Alternatively, if one is familiar with the curves for displacement, velocity and acceleration with respect to time, these can be drawn and the sign for each of them during any time interval can be obtained.

Another method for determining the sign for each of them can be making use of simple calculus.

Equation viz. $x = A \sin \omega t$, $u = \omega A \cos \omega t$ and $a = -\omega^2 A \sin \omega t$ can be used to obtain the desired results.

Analysis of Responses (Problem 2)

Responses obtained from the participants to question number (2), pertaining to direction of acceleration in case of a simple pendulum at different positions can be categorised as follows:

(i) Acceleration is towards the mean position and directed along the path (66.3 per cent).

(ii) Acceleration is towards the mean position and directed horizontally at the given point (15 per cent).

(iii) Acceleration is towards the mean position and tangential at the point under consideration (11.3 per cent).

(iv) Acceleration is towards the suspension point (1 per cent).

(v) No Response (6.3 per cent).

It is thus evident that a number of misconceptions exist regarding the direction of acceleration in case of a simple pendulum. Studies conducted at the international level also indicate the same. One of the basic reasons regarding these misconceptions during the discussion with the participants indicated that motion of the simple pendulum is considered to be a straight line which strictly speaking is not the case. Another important reason is considering acceleration to be proportional to the displacement and oppositely directed which is really the case if the pendulum had been oscillating along a straight line.

Conceptual Approach

To overcome the above situation, following questions are relevant so as to arrive at the correct result for determining the direction of acceleration:

(i) Is the motion of a simple pendulum stately a straight line?

(ii) If not what is the nature of the path executed by a simple pendulum?

(iii) How is acceleration defined?

(iv) Through discussion, it may be inferred that the path of the pendulum is an arc of a circle and the acceleration is the change in velocity per unit time. Then another question of significance can be posed viz, Is the speed of the pendulum constant while executing the motion during the period 0 to T ? Obviously, since the speed is not constant, it may be discussed that magnitude as well as direction of velocity, both, are changing with time and acceleration in that case can't always be towards the suspension point. To understand it further, one may ask to draw a vector (u1) representing velocity at a slightly later time. The direction of difference of these vectors (u1–u) will give the direction of acceleration during the time interval between the two points. It will be seen that the direction of acceleration will be as shown in the Fig. 3 given below:

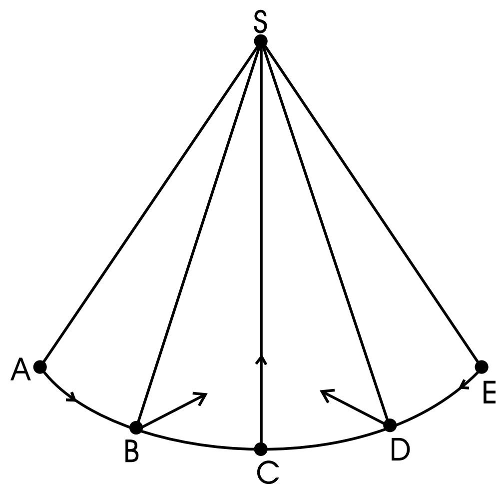

Fig. 3

Suggestions

It is obvious that teachers misconceptions have direct implications on teaching to students. As a remedial measure, it is suggested that before taking up problem-solving, interpretation of a concept or principle should be adequately discussed. The conditions under which the same is applicable should also be thoroughly explained. The concept 'idea first and name after words' is a correct strategy for achieving it. Otherwise, the difficulty of correct interpretation along with complexities in problem-solving will always arise and one will have to deal with both of them for all the times.

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