Using Algebraic Tiles from Secondary Mathematics Kit

P. K. Chaurasia*

Abstract

The Secondary Mathematics Kit developed by National Council of Educational Research and Training (NCERT) describes and elaborates an important and effective pedagogical strategy at the secondary stage, whose potential is rarely exploited yet, which promotes active engagement in subjects, respectively. It arises from a perspective that mathematics is a constructive activity and is most richly learned when learners are actively constructing objects, relations, questions, problems and meanings. In these kits, we show that not only can all learners perform conceptual activities, but the act of construction or activity can engage learners who might otherwise be passive and uninterested. Kit describes a range of practices for producing the examples that generally illustrate, model and demonstrate concepts. We claim that the experiments or activities that learners perform arise from a pool of ideas. Several concepts through these kits can be explored, enriched and extended—a pedagogical focus that is a powerful way of understanding the concepts. Teachers find ways to transfer initiative to learners to experiment with activity or construct mathematical objects, extending their sophistication and deepening their understanding. In this article, we perform the activities based on algebraic tiles along with secondary stage students of KV No. 1, Ajmer. We show that not only all learners can perform conceptual activities, but also the act of construction or activity can engage learners who might otherwise be passive and uninterested. We observe that activities provide ways to reveal learners' depth and breadth of understanding of concepts and general strategies for creating engaging and concept-deepening questions. It is surprising how learners can be energised and intrigued by simple adjustments to standard classroom algebraic tasks.

* *Professor,* Department of Science and Mathematics, Regional Institute of Education, Ajmer–305004

Introduction

The process of learning includes understanding of how a child learns, the nature of the subject matter that we want to teach (or what we want children to learn) and an understanding of what learning is. These three aspects form what we call a model of learning.

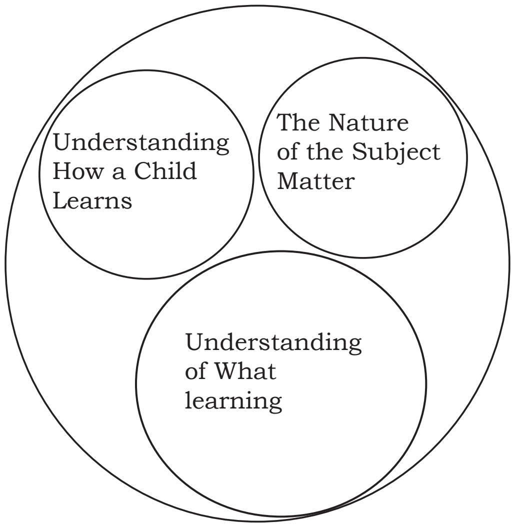


Figure 1. Model of Learning

One of the most important factors that determine the model of learning, that any of us follow is, what we understand by the word 'learning'. Let us consider a sample of ways in which some teachers we spoke to understand learning.

- Learning is production of an expected response to a given stimulus.
- Learning is a change in behaviour due to practice.
- Learning is a change in behaviour due to practice and experience.
- Learning is a permanent change in behaviour because of reinforced practice.

All the definitions above talk of learning having taken place only if it is visible to other people, that is, other people should be able to see the change in behaviour of the learner. For example, according to these definitions, if a child gives a correct answer then the child has learnt; but if the child does not give the correct answer, one will not have learnt it. In this case, it is not a matter of concern as to what understanding the wrong answer shows, or how far has this child developed the concept. So, even though one does not give the expected answer, one may have some feel for the concept. The understanding may not immediately show up in an ability to solve the problem. But as one applies it in more and more situations that one is faced with, one would develop it further.

Some psychologists have taken this view of a child developing an understanding in consideration. They believe that there are other ways of understanding how learning takes place. They do not expect children to immediately display the 'taught' way of solving problems. They respect the need for the child to think and analyse in one's own way, explore and develop one's own way of solving problems. For this, they suggest providing different kinds of tasks to the children, which provide an opportunity to learn. Therefore, instead of a learning environment where learning plays a passive role in the learning process, there is a need to create an environment where the approach to learning should regard the learner as the active agent of one's learning—actively making sense of the physical and social environment around. According to this approach, the learner builds (constructs) one's own understanding based on one's interaction during performing the activities. So the question is how to provoke the students to think harder and about many aspects? How would we get one to explore the concept on one's own? How would we provide the child opportunities where one has to struggle to find one's own methods of solving problems by helping oneself and providing inputs to oneself as and when one needs them? How would we give the child problems that do not have the same kind of solutions, and would expect one to think how each problem can be dealt with? Such innovative practice would require the teacher to discuss with the child what one has done and give oneself an opportunity to solve a wide variety of problems related to the concepts one is trying to learn. A constructivist believes that a child learns by acting on objects. Specially while learning algebra at the secondary stage, we experimented these questions with students working with algebraic tiles. We will see the detail of the activities in later part of the article.

Facilitation of Self-learning Constructivist Environment

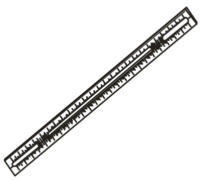
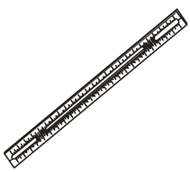
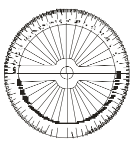
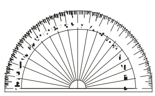
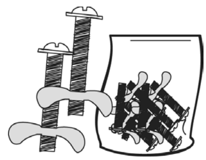
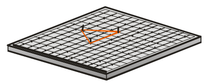
We believe that creating learning by doing environment foregrounds the ways in which teachers can match the above-mentioned opportunities for learning. The integration of hands-on activity-based concrete kits is believed to be very crucial for learning.

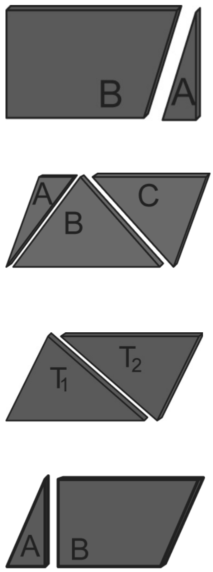
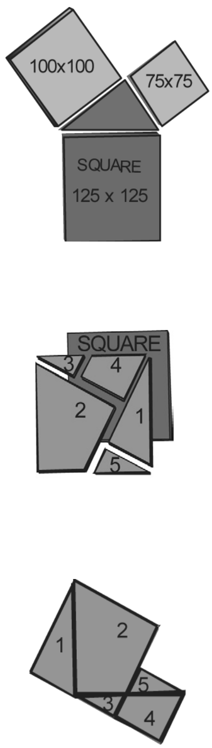
As a matter of practical significance, however, the technologies under consideration are concrete kit items and have some inherent properties that make applying them in straightforward ways generate learning by self-doing constructivist environment. Most traditional pedagogical technologies are characterised by specificity (a pencil is for writing, while a microscope is for viewing small objects); stability (pencils pendulums and chalkboards have not changed a great deal over time); and transparency of function (the inner workings of the pencil or the pendulum are simple and directly related to their function) (Simon, 1969). Over time, these technologies achieve a transparency of perception (Bruce and Hogan, 1998); they become commonplace and, in most cases, are not even considered to be technologies.

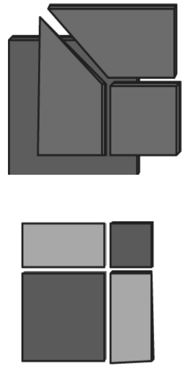
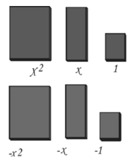
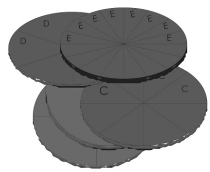
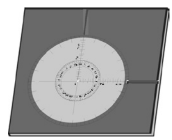
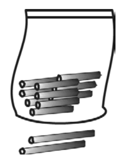
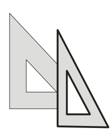
It is believed that when students learn by doing with technology (analog and digital), they may use it as a cognitive tool that helps them to construct meaning, based on their prior knowledge and conceptual framework. Publishers, curriculum specialists, mathematicians,teachers and students have placed a great deal of mathematics and mathematics-related information and activities on the Web. There is a need to consolidate these applications so that students can access a greater range of learning opportunities, and teachers can have a stronger sense of the concrete items or technology's utility and connection to learning outcomes. Technology enhances learning opportunities because it can efficiently support learning by doing, graphing, visualising and computing. Moreover, the technology is used as a medium to provide resources and learning situations that would otherwise be unrealistic or impossible to create.

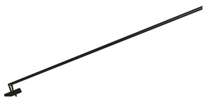
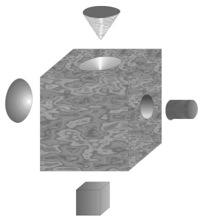
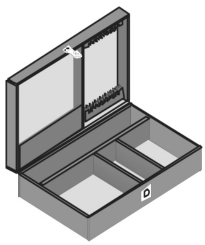
Keeping in view this visionary attitude, the author has developed 'Secondary Mathematics Kit' at NCERT, New Delhi, focusses on broad objectives of 'how to design and develop technology-based content using concrete kit along with different subject-specific open source softwares on various concepts of mathematics as technological interactive content.' Algebraic tiles are one of the tools of this kit and we are focussing on the activities based on those.

Secondary Mathematics Kit (Technical Specification)

S.No	Item Name	Figure/stapes
1.	Plastic Strip (A Type)	
2.	Plastic Strip (B Type)	
3.	Full Protractor (360°)	
4.	Half Protractor (180°)	
5.	Fly nut and Screw	
6.	Geo-board (rectangular)	
7.	Geo-board Dowels	
8.	Rubber bands	

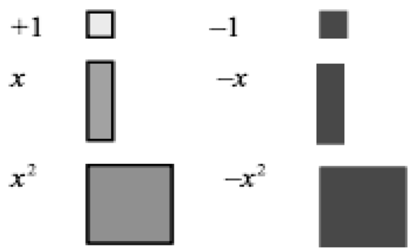
9.	Cutouts : Triangle trapezium b) Two congruent triangle c) A parallelogram and 3 cutouts of a triangle d) Pieces of a trapezium	
10.	Pythagoras Theorem Square With 5 Cut Outs	

11.	Algebraic Identities	
12.	Algebraic Tiles a) x^2 , x , 1 b) $-x^2$, -x , -1	
13.	Cut Outs For Area Of A Circle	
14.	Trigonometric Circle Board	
15.	Connectors For (Strip)	
16.	Connectors (T-type)	
17.	Set Square	

18.	Rotating Needle	
19.	Cutouts With Cuboid Cone Cuboids Cylinder Hemisphere	
20.	Kit Box along with carton	

Algebra Tiles: Tools for Understanding

To help teachers rise to teaching challenges and opportunities for learning algebra, we experimented with algebraic tiles. The introduction of algebra tiles and other manipulatives into the classroom provides mathematics teachers with exciting opportunities to empower students of all learning styles. Through hands-on activities, students become familiar with the uses and applications of algebra tiles. They become comfortable using algebra tiles in their classrooms. And with the experience, users will learn new applications. The activities can be customised to accommodate different needs and levels of understanding. Teacher can select the activities that best meet their specific time constraints and professional requirements. One can introduce the session using any of a variety of ideas. There is a variety of activities designed to address the curriculum goals and objectives. Teacher needs to create a reasonable schedule for her session, being sure to allow time for exploration and questions. Place algebra tiles on the table as follows:



Have students spread the algebra tiles on their worktables and examine them.

Discuss the colours and shapes of the different algebra tiles.

Ask questions such as:

What do you notice about all the negative tiles, –1, –x, and $–x^2$?

Answer: All the negative tiles are red.

Have students stored the tiles in a corner of their worktables so they will have room to make models in the center of their tables.

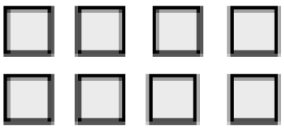
Inform students that they have to record their findings by drawing the models and writing the number sentences and equations they will create in the activities.

Activity 1: Adding Integers

Use your algebra tiles in the steps below to illustrate the addition of two positive numbers — 5 + 3

Have students show two groups of positive tiles. In one group, model +5. In the other group, model +3.

Ask: How can we model 5 + 3 with these tiles? Elicit the fact that to add the two groups, they should be moved together.



Ask: What number sentence describes the model?

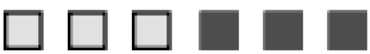
Answer: 5 + 3 = 8

Since the sum of a number and its opposite is zero, together, a positive tile and a negative tile represent zero and are called a zero pair. Use your algebra tiles to model a zero pair.

Ask: What number sentence describes this zero pair?

Answer: 1 + (–1) = 0

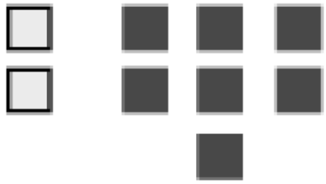
Have students model 3 + (–3) at their desks while you model. Model 3 with 3 yellow tiles and model –3 with 3 red tiles.



Ask: What number sentence describes the model?

Answer: 3 + (–3) = 0

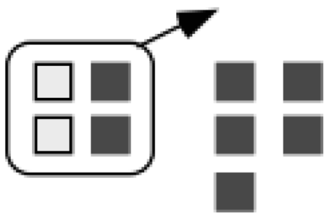
Have students model 2 + (– 7) at their desks while you model the expression on the overhead projector. Model 2 with 2 positive tiles and –7 with 7 negative.

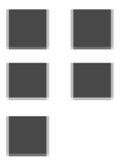


Ask: What expression does this model represent?

Answer: 2 + (–7)

Have students join the two groups of tiles. Match pairs of positive and negative tiles and remove them. Elicit the fact that you can remove zero pairs because their value is zero. Elicit the fact that you cannot form any more zero pairs because all yellow tiles have been used.





Ask: After removing the zero pairs, what tiles are left?

Answer: 5 red tiles, representing –5

Ask: What number sentence describes the model?

Answer: 2 + (–7) = –5

. Challenge students to find another sum

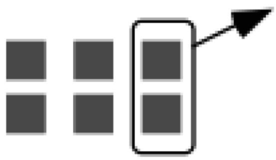
a. 4 + (– 9) b. – 3 + (– 8) c. 9 + (– 3)

Activity 2: Subtracting Integers

Use algebra tiles on the overhead projector as per the steps below to illustrate the subtraction of two integers.

Say: We can model – 6 – (–2).

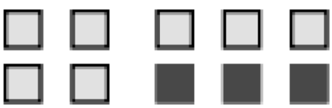
Start with 6 negative tiles. To subtract –2, remove 2 negative tiles.



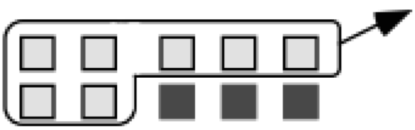
Ask: What number sentence describes the model? Answer: –6 – (–2) = –4

Say: We can model 4 – 7.

Have each group place four positive tiles in a group. Ask them to add zero pairs until they have seven positive tiles in the set.



Have the students remove seven positive tiles.



Say: This is one form of subtraction.

Ask: How many tiles are left?

Answer: Three red tiles, representing – 3.

Ask: What number sentence describes the model?

Answer: 4 – 7 = – 3

Ask students use algebra tiles for the following subtractions. After each group has to record its models and number sentences, uncover the 'Model/Answer' column.

a. 2 – 6 b. 2 + (–6) c. –3 – 8 d. –3 + (–8)

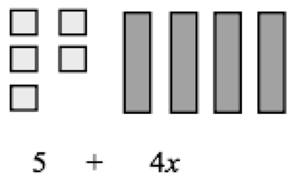
Elicit discussion about the differences and sums.

Activity 3: Simplifying Algebraic Expressions

Write this expression on the board: 5 + 4x

Ask: How can we model this expression?

Give students an opportunity to respond. Then model the expression with the group.

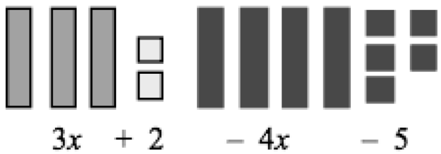


Write this expression on the board: 3x + 2 – 4x – 5

Say: Before we model this expression, remember that subtracting is the same as adding the opposite, so we can write the expression as 3x + 2 + (–4x) + (–5).

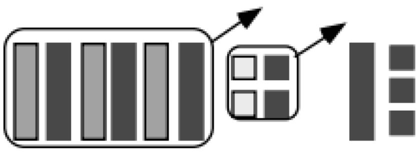
Ask: How can we model this expression?

Give students an opportunity to respond. Then model the expression with the group.



Ask: How can we simplify the expression?

Elicit the fact that simplifying means collecting like terms (like tiles) by using zero pairs.



Remove zero pairs of x tiles and zero pairs of integer tiles.

Say: After we move aside the zero pairs, the simplified expression is left.

Answer: 3x + 2 – 4x – 5 = –x – 3

Have students use algebra tiles to model and simplify the following expressions. After each group has recorded its models and expressions, uncover the 'Model/Answer' column.

a. 4x + 8 – 3x b. 5x – 9 – 2 – 3x c. –3x + 7 + x – 6

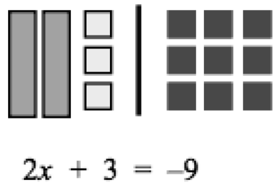
Activity 4: Solving Linear Equations

Write this expression on the chalkboard:

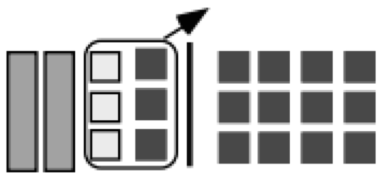
Write: 2x + 3 = –9

Ask: How can we model and solve this equation?

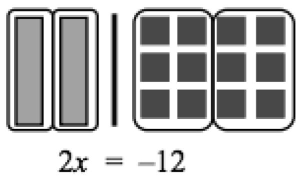
Give students an opportunity to respond. Then manipulate the tiles while you explain each step. 2x + 3 = –9



Add three negative tiles to each side to create zero pairs on the side with the x-tiles.



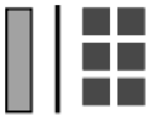
Remove zero pairs to show 2x = –12.



Say: We want to get x alone for a solution. First, we can make two groups of equal numbers of tiles on each side of the bar. Then we can remove one set of the tiles from each side of the bar. Remember that whatever we do to one side of an equation, we must do to the other side. 2x ÷ 2 = –12 ÷ 2

Ask: What is the solution?

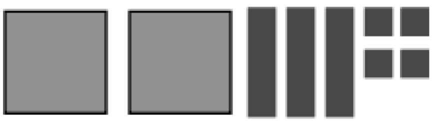
Answer: $x = -6$



Activity 5: Modelling Polynomials

Place the tiles on the table.

Ask: What expression does this model represent?



Answer: $2x^2 - 3x - 4$

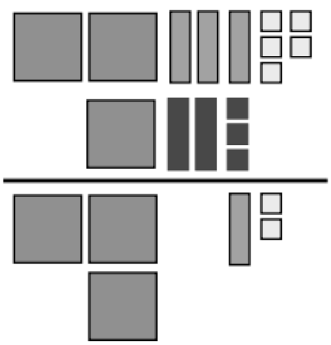
Activity 6: Adding Polynomials

Remind the students that they can model addition of polynomials by modelling the two polynomials, joining them, and removing zero pairs.

Ask: How can we model the addition $2x^2 + 3x + 5$ and $x^2 - 2x - 3$?

Accept all reasonable answers. Then model the addition with tiles on the overhead projector. Call attention to the zero pairs.

Ask: What is the sum?



Answer: $3x^2 - x + 2$

Activity 7: Subtracting Polynomials

Remind students that they can model subtraction of a polynomial by adding the model of the inverse of the polynomial to be subtracted to the model of the first polynomial.

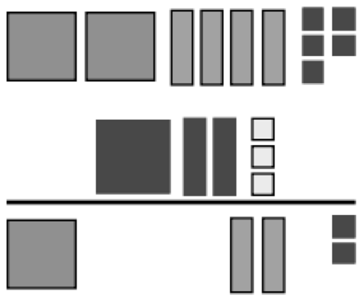
Ask: How can we model this subtraction?

$2x^2 + 4x - 5 - (x^2 + 2x - 3)$

Accept all reasonable answers. Then model the addition with tiles.

Build a model for $2x^2 + 4x - 5$.

Subtract $x^2 + 2x - 3$ by adding its opposite, that is $-x^2 - 2x + 3$. Model this expression.



Answer: $x^2 + 2x - 2$

Combine models and remove zero pairs to model the result.

Ask: What expression does the resulting model represent?

Activity 8: Multiplying Polynomials

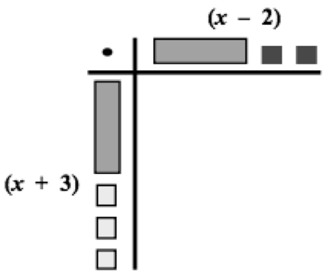
Ask students how to show basic multiplication facts.

Ask: How can we find the product 3 and 4?

Ask: What is the product of 3 and 4?

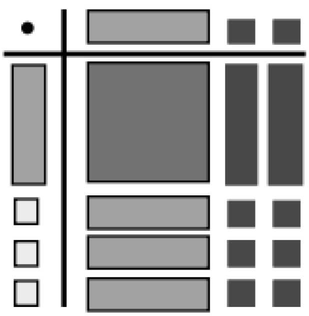
Help students to relate a basic multiplication table to a rectangular array to model the multiplication of polynomials.

Ask: How can we model the multiplication of $(x + 3)(x - 2)$?
Elicit the fact that algebra tiles that represent the first factor $(x + 3)$ are placed on the vertical axis and algebra tiles that represent the second factor $(x - 2)$ are placed on the horizontal axis.



Ask: How can we model $(x + 3)$
$(x - 2)$?

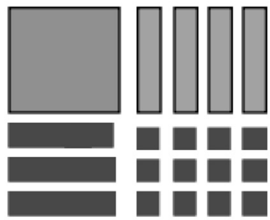
Give students a chance to respond and model the multiplication with algebra tiles.



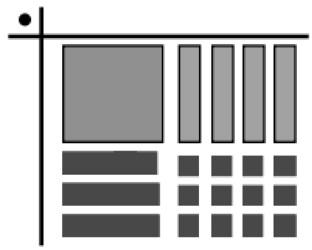
$(x + 3)(x - 2)$

Ask: What is the product of $(x + 3)$
$(x - 2)$?
Answer: $x^2 + 3x - 2x - 6 = x^2 + x - 6$

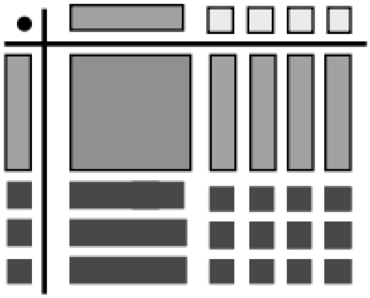
Activity 9: Factoring Polynomials
Show the model



Call attention to the fact that the model shows a rectangular array with the tiles arranged in descending order.
Ask: What expression does this model represent?
Answer: $x^2 + 4x - 3x - 12$
Challenge students to suggest how the expression can be factored by using algebra tiles. Give students a chance to respond and then demonstrate how to factor a polynomial.



Build an axis around the rectangle.
To factor the polynomial, find the dimensions of the rectangle. Determine which tiles should be placed on the horizontal axis and which tiles should be placed on the vertical axis. Call attention to the fact that all negative tiles should be placed on the same axis.



Ask: What expression does the resulting model represent?
Answer: (x – 3) (x + 4)

Summary

Thus, we have looked pedagogical approaches in using Algebraic tiles. Algebraic tile tools based interactive content materials will form a resource pool and motivate users from exploratory mode towards expressive mode. It has been observed that students generally enjoyed using the manipulative and this enjoyment was consistent among all groups. Normally groups did not differ in achievement using this manipulative based instructional strategy. Students have shown confidence that their level of understanding was enhanced by using concrete materials. This appeared to be true for all learning style groups. Students appeared to be able to apply the knowledge they had acquired from concrete experiences in abstract situations.

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