Mathematics has to be Taught the Way Mathematics is

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Abstract

Mathematics is a study of sets, where a set represents a property which is either true or false. One of the vastest areas of world of contemplative beauty is Mathematics. This alone is a sufficient reason for study of Mathematics (King, 1997). There are people who perceived Mathematics as dogmatic, inflexible and dull. But the fact is that the basic elements of Mathematics are rare combinations: logic and intuition, analysis and construction, generality and individuality, precision and beauty. Harmony (order) and non-discriminatory approach (for every), in Mathematics emphasised the value of peace in societies. The Education Committee (Kothari 1964–66) recommended Mathematics as a compulsory school subject for all school students. NCF-2005 emphasises that Mathematics should be visualised as a vehicle to train a child to think, reason, analyse and articulate logically. There are instances where Mathematics is taught differently—fundamentals are not given due importance, mathematical results are conveyed through authoritative communication, proofs and theorems are replaced by illustrations or inductive arguments (solving only routine problems with main emphasis on drill). Keeping these in mind, in this article we discuss some innovative methods of teaching Mathematics which are some combinations of: Venn Diagrammatic Approach, Mathematical Modeling, Mathematics Lab, Problem Solving (including non-routine problems), Index of Teaching, Action Research in Mathematics, Feeling and Aesthetics in Mathematics, Historical Approach.

Introduction

Mathematics is essentially a study of sets and their interrelations, where, a set represents a well-defined property – a property which is either true or false. Mathematics links abstract ideas to physical things; and creations in mathematics are

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observable and provide aesthetic experience. "One of the vastest areas of world of contemplative beauty is mathematics. This alone is sufficient reason for study of mathematics" (King, 1992). There are people who perceived mathematics as dogmatic, inflexible, and dull. But the fact is that basic elements of mathematics are rare combinations, viz., logic and intuition, analysis and construction, generality and individuality, precision and beauty. Further harmony (order) and non-discriminatory approach (for every) in mathematics emphasises the value of peace in societies.

National Curriculum Framework, 2005 (NCF, 2005) emphasises, "Mathematisation (ability to think logically, formulate and handle abstractions) rather than 'knowledge' of mathematics (formal and mechanical procedures) is the main goal of teaching of mathematics".

Keeping these fundamental aspects of mathematics in mind, any method or strategy of teaching-learning of mathematics has to be evolved. There are instances where mathematics is taught differently — fundamentals are not given due importance, mathematical results are conveyed through authoritative communication without a justification, proofs of theorems are replaced by illustrations, indicative arguments, or 'empirical induction', solving only routine problems with main emphasis on drill.

Here, we discuss some of the innovative methods of teaching mathematics which are a combination of – Venn diagrammatic approach, mathematical modeling (application of mathematics), mathematics lab, problem solving (including non-routine problems), action research in mathematics, historical approach, feeling of aesthetics in mathematics.

Ultimately any teaching-learning of mathematics has to provide an active experience in Mathematics.

Mathematics education is mainly concerned with teaching-learning of mathematics. For effective and meaningful teaching-learning of mathematics, mathematics education has to seriously deal with constructive invention, motivating intuition, applications and aesthetics within the framework of 'deductive form of mathematics'. There is a poor response to mathematics education at higher education level. Mathematics education at school level has to answer, "does the content of mathematics and associated methodology stimulate students to continue in mathematics courses?"

There is hardly any discipline of study without the numbers. The Education Commission (Kothari 1964–66) recommended mathematics as a compulsory subject for all school students. Thus, mathematics enjoys unique status in a school curriculum. The National Policy on Education (1986) also emphasises that mathematics should be visualised as the vehicle to train a child to think, reason, analyse and articulate logically. Apart from being a specific subject it should be treated as concomitant to any subject involving analysis and reasoning. And yet many school students find difficulty with learning of mathematics and fail in mathematics. A major reason for the failure is that the practitioners of curriculum quite often forget the basic assumptions: there is no learning without fundamentals, mathematics should be taught the way mathematics is.

There is a huge gap between prescription and practice of a mathematics curriculum. Most of the time, mathematics classroom is preoccupied with routine teaching and not much time is devoted to learning of mathematics. Hardly a student asks questions in a mathematics classroom. The teacher-training colleges in India prepare the mathematics teachers at secondary level, and paradoxically most of the teacher-training colleges do not have teacher educators with mathematics as a subject at their degree level, or experience of teaching mathematics at school level. Many of the teachers do not distinguish between teaching of mathematics and teaching of science, and often-inductive arguments replace proofs of theorems. Many of the mathematics teachers at secondary level do not understand mathematics, as is evident from the fact that more than 90 per cent in in-service programmes conducted for teachers at RIE Bhopal and Mysore during 1998–2000 did not answer correctly.

(i) Why is it that the product of two negative numbers is positive?

(ii) What is the number after ½?

(iii) Why cannot addition replace multiplication?

A mathematics teacher has to love mathematics, understand mathematics and believe that mathematics is important.

Activities and researches in mathematics education at higher level are almost nil. Vision 2020 for School of Mathematics, Tata Institute of Fundamental Research, TIFR (Paranjape, 1995) also states "The job of teaching and exposition is one area where TIFR has not contributed much as yet. One of the disturbing aspects of mathematics education in India, and also the rest of the world is that of the lack of mathematical sophistication in the education provided to non-mathematicians. Most of the mathematics taught to non-mathematicians centers around the development of the previous century."

Keeping these in view, some of the important issues that promote innovations in teaching-learning of mathematics at school level are the following:

Proofs

"Mathematics is the study of assertions of the form 'p implies q' where p (assumption) and q (conclusion) are each statement about objects that live in mathematical world" (Bertrand Russel, 1917). A proof in mathematics is a deductive process that connects assumption (p) to conclusion (q) by a logical reasoning. "A proof is a construction that can be looked over, reviewed, verified by a rational agent and the mathematician surveys proof in its entirety and thereby comes to know the conclusion" (Thomas Tymoczko, 1979). But there are distortions for the concept of proof. One of the main reasons for the distortions is due to undue importance given to functional mathematics. Quite often, inductive arguments replace proofs. For example, 3,5,7 are first three consecutive odd numbers and primes, but every odd number is not prime. Similarly, the theorem that the sum of the angles in a triangle is 180° cannot be proved by actually measuring the angles of the triangle. If $F(n)=2^n+1$, F(1)=5, F(2)17, F(3)=257, F4=65537 which are all primes but F(5)=641.6700417 is not prime (Euler,1732). Proof requires generality.

Appel and Haken of the University of Illinois proved using a computer the famous Four Colour Conjecture which remained open for 124 years. They produced a new kind of proof which many pure mathematicians have taken with a pinch of salt. What if the computer erred?

"Appel and Haken may have given us a glimpse of the future. A future in which deep theorems routinely will rely for their proofs on the checking of millions of special cases by high speed unmonitorable computers and I gather it is a future they welcome. It is a future without elegance, a world of disfigured mathematics. Truth may choose to live in that world, but beauty will not" (Kingh, 1992)

Now, let us find an answer to the question: Why is that the product of two negative numbers is positive? The proof is unique, interesting and elementary. But what is puzzling is that many students learn the fact by authoritative communication from teacher to student. The proof is as follows: For any two real numbers a, b, let

X=ab+(-a)(b)+(-a)(-b)

(i) X=ab+(-a)(b)+(-a)(-b)

(Distributive Law)

=ab+(-a).0(Additive Inverse)

=ab+0=ab. (z.0=0, and identity)

ii) X=(a)+(-a)b+(-a)(-b)

=0.b+(-a)(-b)

=0+(-a)(-b)=(-a)(-b)

(i) and (ii) imply

ab=(-a)(-b)

Proofs provide an excellent mathematical memory— a memory due to understanding generalisation, formalised structures, logical schemes.

We need to encourage students to give alternative proofs. For example, the statement: 'the sum of first n natural numbers is n (n+1)/2' can be proved by mathematical induction or by writing the series in increasing and decreasing fashion and adding the series (Gauss's way). Some of the students could also try to prove this by contra positive method.

In short proofs in mathematics are fundamental components of mathematics education and they form 'quality' of mathematics education.

Aesthetics in Mathematics

The famous four: Cognitive (Truth), Metaphysical (Reality), Ethical (Justice), Aesthetical (beauty) are great classical components of Philosophy (of liberal education and core curriculum). These four fundamental issues establishes progress and procedures toward quality education.

Aesthetics (beauty) is an integral part of quality in education, and yet many of our educational processes do not include the aesthetics part. Aesthetics is undoubtedly one of the highest qualities of life.

While aesthetics is mainly derived from infinite collections (combinations-patterns), it is difficult to find examples of infinite collections from the real physical world. The natural numbers (counting numbers), which are the first experience in mathematics, is an excellent example of an infinite set. Mathematics is full of infinite sets and excitement. There is a deep sense of aesthetic pleasure that one can derive from mathematics (of course, art, music, and literatures are common source of aesthetics). In fact, mathematicians do mathematics for aesthetic reasons. The proofs (as evidence) in mathematics quite often create excitement (as they exhaust infinite possibilities) leading to aesthetic pleasure.

We quite often study mathematics in a routine way, which is dull, and without any excitement. For many of the mathematics students, the difference between Riemann integral and anti-derivative is that one of them is evaluated between two limits. They do not see the Fundamental Theorem of Calculus as a relation between 'area' and 'tangent' (two un-seemingly related concepts). In this connection, King's (1992) recollection of his college calculus experiences is worth noting: "One full year passed after elementary calculus before we learned the true relation between Riemann integrals and ant-derivatives. We discovered that the connection between these very different notions lies at the very heart of the subject, and that it is one of the genuinely great creations of human intellect. We saw that the connecting argument is a thing of great beauty. Suddenly, we understood that mathematics has an aesthetic value as clearly defined as that of music or poetry."

One of the vastest areas of the world of contemplative beauty is mathematics. This alone is sufficient reason for study of mathematics (King, 1992).

Mathematics possesses not only truth, but supreme beauty— a beauty cold and austere like that of a sculpture without appeal to any part or weaker nature, sublimely pure and capable of stern perfection such as only the greatest art can show (Bertrand Russel).

Despite an objectivity that has no parallel in the world of art, the motivation and standards of creative mathematics are more like those of art than of science. Aesthetic judgments transcend both logic and applicability in the rankings of mathematical theorems: beauty and elegance have more to do with the value of a mathematical idea than does either strict truth or possible utility (Lynn Steen).

The ideas brought forth from the unconscious and handed over to the conscious invariably possess the stamp of mathematical beauty (Poincare).

Aesthetic Experience' variables (George David Birkhoff, 1956)

Birkhoff identifies three typical "aesthetic experience" variables: the complexity (C) of the art object, the harmony or order (O) of the object, and the aesthetic measure (M) of the object. He asserts that these variables are related by the basic formula.

$M=O/C$

(If complexity is less and order is better, then the aesthetic measure is better. Further, if complexity tends to infinity, then the aesthetic measure tends to zero).

Principle for Aesthetic Quality *(King, 1992)*

King proposes two principles, which gauge aesthetic quality of a mathematical notion.

Principle of Minimal Completeness

A mathematical notion N satisfies this principle provided that N contains within itself all properties necessary to fulfill its mathematical mission, but N contains no extraneous properties.

Principle of Maximal Applicability

A mathematical notion N satisfies this principle provided that N contains properties, which are widely applicable to mathematical notions other than N. (here a notion means theorem proof, equation in-equation or definition).

For example, the process of division 64/16=4 (canceling the 6's) appear to be neat. Yet it has no mathematical value— aesthetic or otherwise, because the method has no applicability beyond itself. Euclid's proof of infinitude of prime numbers (n=1 + p1p2 ...pn) satisfies both the principles (the primes are not restricted and have wider applicability in the theory of numbers and the proof is complete in itself). Similarly, Pythagorean proof of irrationality of J2 satisfies both principles. Thus, both of these are 'elegant' and have aesthetic quality.

Mathematical Modeling

A mathematical model is a simplified mathematical representation of a real situation with a mathematical system (a model is something which represents something else). Although a real situation involves a large number of variables and constraints, usually small fraction of these variables and constraints that truly dominates the behaviour of the real system, and identification of such variables and constraints is one of the purposes of mathematical modeling.

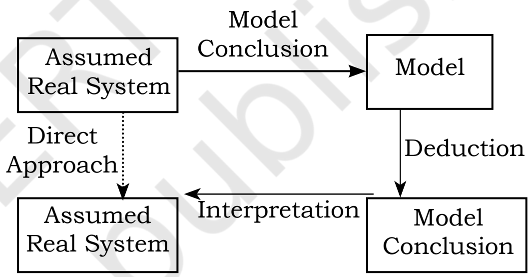
The set of natural numbers with usual addition and multiplication form a good mathematical model of real situations concerned with counting process. Vectors are excellent mathematical models that predict and explain many physical phenomena with perfect accuracy. The concept of direction, which is so vague in the physical world is precisely explained by identifying the concept of vector as that of location or coordinate system (Such an identification is guaranteed by the famous result that every finite dimensional vector space is isomorphic to Euclidean space R”). Graphs, (Networks) as mathematical models, cover a wide range of real situations, as they are highly generalised algebraic structures.

Mathematical models are normally thought of as an instrument for selecting a good course of action from the set of courses of action that is covered by the model (here a course of action could be a strategy of selecting a content or some such thing). However, the models have another very important use: they can be used heuristically (that is an instrument of discovery). They provide an effective tool with which one can explore the structure of a problem, and uncover possible courses of action that were previously overlooked. For example, vectors as models have led to discovery of several outstanding and useful results in the vector space theory. The models concerned with drawing of implication diagram (Venn diagram) of given concepts give rise to some very interesting conjectures and their solution later.

Process of Modeling

The process of modeling is depicted in the following figure.

The first step is formulation of the model itself. This step calls for identification of assumptions that can and should be made, so that the model conclusions area as accurate as expected. The selection of the essential attributes of the real system and omission of the irrelevant ones require a kind of selective perception which is more an art than a science,



and which cannot be defined by any precise methodology.

The second step is to analyse the formulated model and deduce its conclusions. It may involve solving equations, finding a good suitable algorithm, running a computer program, expressing a sequence of logical statements, whatever is necessary to solve the problem of interest related to the model.

Mathematical modeling plays a great role in teaching Mathematics

Some of the most important components of teaching a concept in mathematics are:

(i) Motivation for the concept

(ii) Simplification of the concept

(iii) Problem solving

Motivation for learning a mathematical concept may be within the mathematics itself or outside the mathematics and a real-world situation. For instance, it is very difficult to choose an example of an infinite set from a real-world situation; so, in such a situation, the set of natural numbers can be taken as a motivating factor for the concept of 'infinite sets'. On the other hand, a great deal of real world motivates and exemplifies several concepts like vector, derivative, integral, etc.

By simplification of a concept C, we mean breaking of the concept C into simpler sub concepts, or more precisely, it is an identification of meaningful restrictions on C such that Ci (the restricted C) has a simpler characterisation than that of C. Once a concept is simplified into Co, C2.., Cg one is naturally tempted to find various inter-relations among the sub concepts Ci, and that is how the concept C in particular and mathematics in general become richer.

Problem solving heavily depends upon the data of the problem (conditions or assumptions of the problem) and formulation of the problem (mathematical modeling). Mathematical modeling cuts across all the above three components, and each in turn is greatly facilitated by mathematical modeling.

Content in mathematics can be analysed into content proper (what to teach) and its inner organisation, the latter being most closely related to teaching methods.

Teaching methods can be analysed into presentation of the subject matter (use of mathematical models, etc.) and organisation of classroom work, the former being most closely related to content and mathematical modeling. The analogue model of this para is as follows:

Venn Diagrams

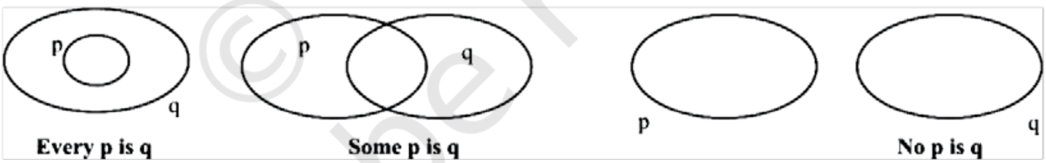
Mathematics is a study of sets where a set is identified with a 'precise property', a property that is either true or false, not both. But strangely the sets do not find the right kind of importance in the teaching-learning process of mathematics in school education. In the process of making mathematics more functional, the most fundamental element

of mathematics "Set" is not used properly as it should have been.

John Venn introduced Venn diagrams in 1880. A Venn diagram represents pictorial inter-relations among sets (well-defined properties), each of which is denoted by a closed region without holes. Though there are other diagrams like line diagram directed graph, etc., to illustrate relationships, Venn diagram has an advantage of space over the others. Given two well-defined properties p, q, the possible relations between them can be represented by Venn diagram in one of the following ways.

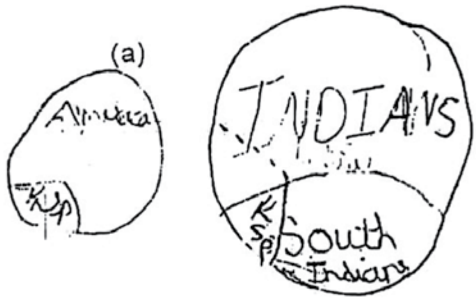
Venn Diagrammatic Representation Approach (VDRA) could best be employed in

- better understanding of mathematics because of their visual effect;
- providing clarity in teaching-learning of mathematics;



- finding inter-relations among mathematical properties holistically and accurately, and better analysis of the properties; and
- "Conjecturing" as a consequence of natural creation of some new portion in Venn diagram.

An example of creativity shown by the child (Jagdish of Grade V of DMS, Mysore).



While transacting mathematical content, VDRA is quite helpful in making teachers' ideas clear and also helps in vivid presentation of the content by reducing the verbal statements. It is necessary for them to focus on different components of VDRA (both separately and collectively) giving adequate opportunities for the students to make mathematical representations and think all plausible relationships while solving any of the mathematical problems.

(For more details see the use of Venn Diagrams in teaching-learning of Math, by G. Ravindra School Science, September 2002).

Mathematics Laboratory

Mathematics Laboratory is a place where some of the mathematical activities are conducted. Mathematics labs promote learning by doing and popularise mathematics. Mathlab could be an excellent vehicle of pedagogy of mathematics.

Illustration of some mathematical ideas using a computer or a model, preparation of teaching aids, verification of proofs without affecting deductive nature of mathematics, application of mathematics to other disciplines, etc., can be effectively carried out in math labs. It would be an amazing experience to observe that it is not possible to change the shape of a triangle formed by three sticks loosely fixed at the three corners while the same is not true for shapes of quadrilaterals, pentagon, etc. The students would have real excitement when they recall the theorem that two triangles are congruent (same) when the sides of one of them are equal to the corresponding sides of the other. NCFSE-2000 visualised setting up of math labs in the existing science laboratories and converting the existing science laboratories into science cum mathematics laboratories. NCF-2005 also stresses the need of math labs. Now, there are quite a few schools in the country who have come forward to introduce mathlabs.

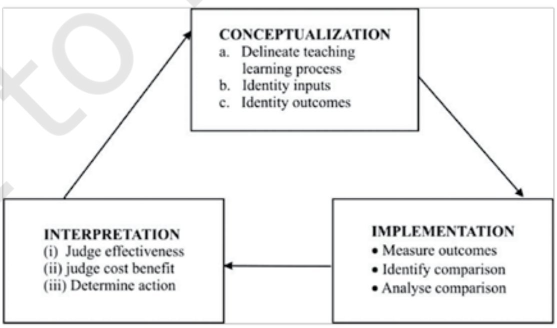
Action Research in Mathematics

Action research is the process of systematically evaluating the consequences of educational decisions and adjusting practice to maximise effectiveness. This involves teachers and school principals, delineating their teaching and leadership strategies, identifying their potential outcomes and observing whether these outcomes do indeed occur. Essentially, action research is examining one's own practice. Action research has the potential to improve practice and provide teachers and principals with deeper understanding of teaching process. At the very least, it provides a process for weighing educational alternatives and making decisions.

Action Research Model

(For details see "Improving Education through Action Research", James E. Mc. Lean Corwin Press INC, A Sage Publications Company, Thousand Oaks, California, 1995)

For example, let us simulate the famous Konigsberg Seven Bridge Problem ("Graphs", Claude Berge, North Holland, 1985) to illustrate



action research in mathematics. The problem is staged as follows.

The city of Konigsberg (today known as Kaliningard) is divided by the Pregel River that surrounds the island of Kneiphof. There are seven bridges in the city as shown in the following figure. Can a pedestrian traverse each bridge exactly once? This problem puzzled the residents of Konigsberg until Euler showed in 1736, that no solution exists.

With the help of the students and using coloured chalk powders, a teacher could draw a huge analogue sketch of the above Fig. 1 on the surface of the school ground, distinctly marking the water of Pregel river by blue chalk powder and the seven bridges replaced by yellow pathways. a, b, c, d would represent the land portions of Konigsberg. Then the play begins. Each of the students makes several attempts to traverse each of the yellow strip exactly once without stepping into the blue portion starting from ‘a’ and back to ‘a’. The teacher permits them to repeat the same from b to b from c to c or from d to d. Then a puzzling question: “why cannot we?” direct them to a solution. The teacher helps each of them to draw the network as shown in the Fig. 2. Perhaps some of them possibly start thinking like Euler, that the odd number of edges at a, b, c, or d creating an obstacle in their traverse. So, the teacher has done his job.

Practice is the hallmark of action research. Mathematical modeling, mathlab, venn diagram, and alternative proofs could effectively be used as action research strategies and processes.

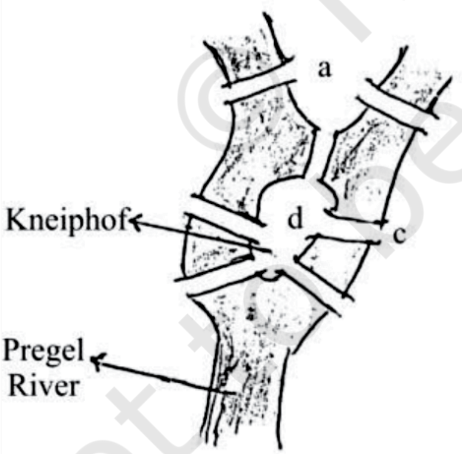
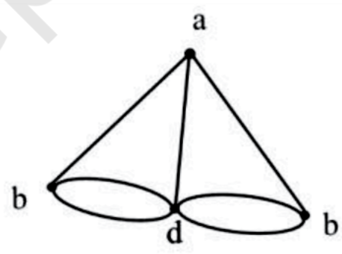


Fig. 1



Network (or graph)
a, b, c, d are land areas and edges are the connecting bridges

Fig. 2

Problem Solving

The ultimate aim of learning mathematics is to be able to solve problems. The problems stated in textbooks are generally "routine problems", which are characterised by mathematical tasks— computational exercises, applying formulas, using algorithms, solving verbal (word) problems. Non-routine problems have no readily available procedures, and generally a solution is obtained by using mathematical concepts. In connection with non-routine problems, Homi Bhabha Centre for Science Education is doing a great job by organising Mathematics Olympiad programmes (including training) for school students. The International Mathematics Olympiads (IMO) has a significant impact on the mathematical education of several participating countries and also on the gifted children and their pedagogical benefit is undeniable. For example, to get a feel of the problems in IMO, we list two of them.

(1985) — A circle has center on the side AB of the cyclic quadrilateral ABCD. The other sides are tangents to the circle. Prove that AD + BC=AB.

(1991) — Let S = {1, 2, ---..., 280}. Find the smallest integer n such that each n-element subset of S contains five numbers which are pair-wise relatively prime.

Though there are different viewpoints regarding instruction in problem solving, one of the most lucid model for problem solving is given by the mathematician George Polya in his book, 'How to Solve It' (Princeton University Press, 1973). His problem solving model is as follows:

First	-	Understanding the problem
Second	-	Devising a Plan
Third	-	Carrying out the Plan
Fourth	-	Looking Back

Index of Teaching

Index of Teaching I(T): the number of questions asked by distinct students in a classroom per period. I(T) is closely related to the capability of a teacher to motivate the child to learn. In an experiment conducted in 10 classrooms of schools of Sandur (Karnataka), I(T) was found to be almost zero, implying almost no Learning. I(T) has to be normally at least 5. I(T) can be self-monitored by the teacher and it can do wonders to promote student-centered learning.

History of Mathematics

Mathematics started growing open endedly from eighteenth century B.C. with the contribution of Egyptian priest Ahmes. His contribution included problems in Geometry and the contributions are treasured in the Rhind collection at British museum, ('The Great Mathematicians' H.H. Turnbull, May 1951).

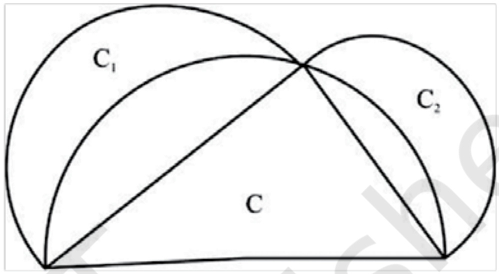
India has a rich mathematical heritage; an instrument was actually used for drawing circles in The Indus Valley as early as 2500 B.C. (Mackey 1938). Aryabhata I (475 A.D.), Brahma Gupta (Seventh century), Mahavira (850 A.D.), Bhaskara II (1150 A.D.), Madhava (Fourteenth century), Ramanujan (1887–1920) have made significant contribution to world of mathematics. Ramanujan is considered to be the greatest mathematician of 20th century.

Through the history of mathematics, one can see how a mathematician thinks and how his imagination helps the 'development of mathematics'. Thus, the history of mathematics has an important pedagogic value. History of great mathematicians and their great creations has to be a part of mathematics curriculum, both at school and higher education level.

Examples

1. In trying to square the circle (i.e., finding as square whose area is equal to that of a given circle), Hypocrates (Greek mathematician during fifth century B.C.) discovered that two moon-shaped figures could be drawn whose areas were together equal to that of a right-angled triangle. Please see the following figure. This was the first example of a solution in quadrature's (i.e., constructing a rectilinear area equal to an area bounded by one or more curves). The sequel to attempts of this kind was the invention of Integral Calculus by Archimedes (287–212 B.C.). Apart from this problem, 'Duplication of the cube', 'Trisection of a given angle' encountered by the Greek mathematicians during fifth and fourth centuries B.C. These three problems eluded solution for several centuries (nearly twenty-five centuries) and ultimately a solution was found in nineteenth century A.D. Surprisingly, the solution was Algebraic, though the problems belonged to geometry.

$C=C_1+C_2$ (Hippocrates, 5th Century BC)



2. As HW Turnbull puts it, Carl Friedrich Gauss (1977–1555) was the last complete mathematician. Gauss pointed out to his father an error in an account when he was three. There was an interesting anecdote associated with Gauss. When Gauss was eight, he and his classmates were asked by their teacher to find the sum of the first hundred natural numbers (the teacher wanted to take some time off from the class and perhaps thought that this could be a teaser to hold them in the classroom till he came back). Gauss instantly wrote the answer as 5050 on his slate and handed it to the teacher, before the teacher could leave the room. Gauss arranged mentally the numbers 1, 2,...., 100 in pairs (1, 100), (2, 99),...,(50, 51). There exactly 50 and the sum of the numbers in each pair is 101. Hence the desired number is 50 times. The whole process took him just a few seconds.

The Prime number theorem states that when n is a large number, the result of dividing n by its logarithm gives a good approximation to the total number of primers less than or equal to n. Gauss knew this and it is not known whether Gauss proved this. This is manifestation of amazing genius of Gauss, and an example of the highest kind of intuition. It took almost a century to prove the statement of Gauss's conjecture. In 1896, Hadmard and Va1lee Poussin proved independently the Prime Number Theorem. This startling result provided a seemingly unrelated connection between the discrete mathematics of whole numbers and the continuous mathematics involved with the logarithm. This is a real joy.

3. Srinivas Ramanujan (1887–1920) was undoubtedly one of the greatest mathematicians of the twentieth century. He was a self-taught genius and highly creative, guided by imagination and intuition of the highest order. "What Mozart was to music and Einstein was to physics, Ramanujan was to mathematics" (Clifford Stok). Many are familiar with the famous taxi-cab number story told by Hardy. "I remember once going to see him when he was lying ill at Putney. I had ridden in a taxi cab no. 1729, and remarked that the number seemed to me rather a dull one, and that I hoped it was not an unfavourable omen. 'No', he replied, 'it is very interesting number; it is the smallest number expressible as sum of two cubes in two different ways". The two ways are: $12^3+1^3=1729=10^3+9^3$. Ramanujan offered an approximation to viz.,

$$= (97\tfrac{1}{2} - 1/11)^{1/4} = 3.1415926526----.$$

What's amasing is 'the way Ramanujan was thinking'. He hardly had any access to any of journals in mathematics. He used to read less and think more. Intuition is hallmark of Ramanujan.

Many of the great mathematicians did their best work when they were relatively young. Hardy made a remark 'Mathematics is a young man's game'. A mathematics teacher will have to understand mathematics and teach young children mathematics the way mathematics is, so that they get interested in mathematics and mathematics grows.

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