Mathematisation of Thought: Right or Wrong Answer in Mathematics Classroom

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Abstract

According to the National Curriculum Framework, 2005, the aim of mathematics teaching is 'mathematisation' of thought, which means to be able to understand the world in the language of mathematics. Research suggests that stereotyped classroom teaching involves a mechanical or procedural approach to problem solving, with emphasis on getting a right answer. The children are hence socialized to approach a problem in procedural manner to get a legitimate correct answer without giving consideration to the realistic context of the problem. This paper studies the approaches of children studying in Class V of a reputed government CBSE school, to realistic mathematics word problems and how these approaches are related to their classroom achievement. The analysis of the responses reveals that the children approach the problems in a procedural manner and their school grades are not related to their realistic understanding of the problem.

Introduction

"The main goal of mathematics in school is the 'mathematisation' of the child's thinking." — National Curriculum Framework, 2005.

'Mathematisation' of thought implies that the child learns to think about the world in the language of mathematics. That is, she becomes capable of mathematical modelling and applying mathematics to solve problems in real life situations.

The major emphasis of NCF, 2005, is on linking a child's everyday life experience to school mathematics. An attempt is made to add contextual details to mathematics in order to link it to the child's everyday knowledge in the form of word problems which form an important part of mathematics curriculum.

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Vision for School Mathematics

- Children learn to enjoy mathematics rather than fear it.
- Children learn important mathematics; Mathematics is more than formulas and mechanical procedures.
- Children see mathematics as something to talk about, to communicate through, to discuss among themselves, to work together on.
- Children pose and solve meaningful problems.
- Children use abstractions to perceive relationships, to see structures, to reason out things to argue the truth or falsify of statements.
- Children understand the basic structures of mathematics: arithmetic, algebra, geometry and trigonometry, the basic content areas of school mathematics. All offer a methodology for abstraction, structuration and generalisation.
- Teachers engage every child in class with the conviction that everyone can learn mathematics (NCF 2005:43)

Word problems are verbal descriptions of a problem situation wherein one or more questions are posed and the answers to which can be obtained by application of mathematical operations to information (usually numerical data) available in the text. The types of word problems used in classroom consist of a text embedded in a real life situation and the answers derived would work in the given situation.

The problems are intended to help the children to develop problem solving abilities and to use the mathematics learnt in classroom in solving problems in real life situations. Thereby showing that the mathematics they learn in classroom will be useful in everyday life, that is; leading to their 'mathematisation' of thought. In contrast, the current practice of word problem solving in classroom does not foster the ability of mathematical modelling in students.

Need of the Study

In classroom situation word problem solving involves learning specific tricks like identifying keywords (e.g. altogether, remaining), applying procedural memory and doing mechanical exercises of applying the four mathematical operations without giving any considerations to realistic problem context and reality constraint. Researchers have shown that children across variety of societies frequently fail to bring realistic considerations in finding solution to word problems (Greer, 1997; Reusser, 1988; Silver et al., 1993; Verschaffel and De Corte, 1997).

De Corte and Verschaffel (2000), in their paper Connecting Mathematics Problem Solving to the Real World state "practice surrounding word problems is controlled by a set of (largely implicit) rules that constitute "word problem game". These rules include the following assumptions:

1. Every problem presented by the teacher or in the textbook is solvable and makes sense.
2. There is only one exact numerical correct answer to every word problem.
3. The answer must be obtained by performing basic arithmetical operations on all numbers stated in the problem.

These largely implicit rules are learnt by a child through her socialisation in the mathematics classroom. The teaching practices socialise a child's approach to word problem solving wherein the implicit rules learnt by them guide their approaches and they solve problems with neither realistic considerations nor the use of their common sense. The teaching practice involves the teacher first presenting what is to be acquired and then the students practising on a set of tasks given to them. This stereotyped approach thus focuses on the procedure of getting the correct answer rather than the concept of the problem; that is the argumentation involved (Saljo and Wyndhamn, 1997; Silver et al., 1993; Verschaffel et al., 1994).

Despite NCF, 2005 recommendations of linking a child's everyday experience to classroom mathematics and the aim of 'mathematisation' of thought, the current mathematics teaching practice remains highly procedural and the aim remains defeated. Children's approach to realistic mathematics word problems is a less researched area in Indian context. This study tries to answer the following questions:

- How do children approach realistic mathematics word problems?
- Are realistic approaches of children to the word problems related to their classroom achievement?

Objectives of the Study

- To study the approaches of children to realistic mathematics word problems.
- To study the relationship between the students' school grades and their realistic understanding of the problem.

Research Procedure: The study is an exploratory field study, based on data obtained from 80 students of Class V of a CBSE school in Varanasi, selected purposively. The chosen school is a reputed one catering to children of government employees of all classes. An arithmetic test consisting of ten word problems was used, of which three were simple problems while remaining seven required realistic considerations therefore called realistic mathematics word problems. The problems were based on the four mathematical operations and were in both English and Hindi.

The test was administered to 80 students of two sections of Class V. Class V-A was told that they were free to answer creatively and give whatever solution they thought was appropriate for solving the problems and there was no right or wrong answer. The second group V-B was not given any such instructions and was simply asked to solve the given problems. Both the groups were further asked to give reasons for not solving a particular problem. No further hint or help was given to the students while they solved the problems. There was no time limit and most of the students took approximately an hour to do the test.

Evaluation of Achievement in School: The school follows a detailed plan to measure and assess students' achievement in arithmetic. It provides grades separately along the following dimensions:

1. Formation and recognition of numbers (FNC)

2. Understanding basic concepts (UBC)
3. Ability to compute (AC)
4. Problem solving ability (PSA)
5. Project
6. Assignment
7. Oral

The overall grade comprised the grades in above seven dimensions. Here the researcher used the grades obtained by the students in problem solving ability.

The conversion of the grades into percentage as used in school is as follows:

A+ — 90%- 100%
A — 75% - 89.99%
B — 56% - 74.99%
C — 35% - 55.99%
D — 0% - 34.99%

Data Analysis and Interpretations

Out of 80 answer sheets one was found incomplete therefore only 79 answer sheets were used for analysis. The responses for the problems were coded as follows:

EA – Expected Answer (when the answer is obtained through procedural approach to the problem).

OA – Other Answer (these are unclassifiable answers, when the child attempts to obtain the answer by using any mathematical operation mechanically without understanding the problem, also a procedural approach. These are wrong answers according to classroom evaluation).

NA – No Answer (when the child has not attempted to solve the problem).

RA – Realistic Answer (when the child has solved a problem by considering the realistic context given in the problem, that is; the approach is contextual).

TE – Technical Error (when the child committed a calculation mistake in solving the problem to get expected answer).

Out of ten problems three were simple, referred to as S1, S2, S3 and the remaining seven referred to as P1, P2, P3, P4, P5, P6, P7; required realistic considerations to solve them (realistic word problems). The analysis of the answer sheets revealed that there was not much effect of the differences in instructions given on the approaches of the students to solve the problems. The type of responses given by the two groups combined to the ten problems is given in percentage as follows:

Table 1
Percentage of different types of responses to the ten problems given by both groups

	S1	S2	S3	P1	P2	P3	P4	P5	P6	P7
EA	100%	46.8%	55.7%	84.8%	17.7%	68.4%	31.6%	8.9%	50.6%	43%
OA	-	46.8%	41.8%	5.1%	55.7%	21.5%	35.4%	87.3%	40.5%	43%
NA	-	6.3%	2.5%	2.5%	19%	2.5%	3.8%	2.5%	5.1%	10.1%
RA	-	-	-	6.3%	3.8%	2.5%	6.3%	0%	0%	0%
TE	-	-	-	1.3%	3.8%	5.1%	22.8%	1.3%	3.8%	3.8%

It is evident from Table 1 that the maximum percentage of students gave an expected answer or some other answers to the simple as well as realistic word problems. The other answers usually involved children applying any mathematical operation on the numbers given in the problem in order to solve it. To simple problem S1, 100% responses were expected answers. To problems S2 and S3, 46.8 % and 55.7% children gave an expected response, respectively and 46.8% and 41.8% gave other answers respectively. To the realistic problems also, the maximum percentage of students either gave an expected answer or gave other answers. A very small percentage of students gave realistic responses to problems P1, P2, P3 and P4 while no student gave any realistic answer to problems P5, P6 and P7.

The percentage of children from different grade groups giving realistic answers to the problems P1 to P7 from two different groups is given as follows:

From the Table 2 and 3 it is clear that the percentage of realistic responses in both the groups is very less. In Group 1 just 20% children from grade group C and 3.7% children from grade group D gave realistic responses to problem 1. Apart from P1 none of the children in Group 1 gave realistic response to the remaining problems P2 to P7. Similarly in Group 2 children gave no realistic response to problems P5, P6 and P7. It can be seen from the table that realistic responses do not depend on the grades achieved. That is, very small percentage of high achievers (A+ and A), the average achievers (B) and the low achievers (C and D) took a realistic approach to the problems in group 2 while in group 1 all children from all grade groups (except grade group C and grade group D to P1) did not make any realistic considerations to solve any of the realistic word problems, showing that even those who are good at problem solving don't see the realistic aspect of a problem.

Table 2
Percentage of Realistic Responses of Group 1 (with instruction)

	P1	P2	P3	P4	P5	P6	P7
A+	0%	0%	0%	0%	0%	0%	0%
A	0%	0%	0%	0%	0%	0%	0%
B	0%	0%	0%	0%	0%	0%	0%
C	20%	0%	0%	0%	0%	0%	0%
D	3.7%	0%	0%	0%	0%	0%	0%

Table 3
Percentage of Realistic Responses of Group 2 (without instructions)

	P1	P2	P3	P4	P5	P6	P7
A+	16.7%	16.7%	16.7%	33.3%	0%	0%	0%
A	0%	33.3%	0%	0%	0%	0%	0%
B	0%	0%	25%	0%	0%	0%	0%
C	25%	25%	0%	25%	0%	0%	0%
D	4.2%	0%	0%	8.3%	0%	0%	0%

Table 4
Percentage of Expected Answers and Other Answers on Realistic Problem (Both groups)

	P1		P2		P3		P4		P5		P6		P7	
	EA	OA	EA	OA	EA	OA	EA	OA	EA	OA	EA	OA	EA	OA
A+	87.5	0	12.5	62.5	87.5	0	50	25	25	75	50	25	87.5	12.5
A	100	0	33.3	33.3	66.7	0	100	0	33.3	66.7	100	0	66.7	33.3
B	100	0	25	37.5	75	0	62.5	12.5	12.5	87.5	100	0	62.5	25
C	66.7	11.1	11.1	66.7	66.7	22.2	44.4	22.2	11.1	77.7	33.3	66.7	44.4	44.4
D	84.3	5.9	17.6	56.9	64.7	29.4	17.6	45.1	3.9	92.2	43.1	47.1	31.4	51

Analysis of all the realistic problems reveals that maximum number of children from all grade groups gave an expected answer or other answers by mechanical applications of arithmetic algorithms without understanding the problem. Where the children had problems dealing with fractions (P2), division of big numbers (P4) and conversions (P5) the number of children giving other answers increased for all grade groups. The analysis reveals that all the children approach a problem in a procedural manner with an aim of getting a numerically correct answer required in the classroom.

P2 was a simple fraction problem stated as: Ankur has bought 4 planks of 2□ m each. How many planks of 1m each can he cut out of these planks? 10 planks was the most often given answer in response to the above problem. Maximum students from all grade groups gave other answers in which children attempted to obtain the answer by using any mathematical operation mechanically without understanding the problem. Apparently, it involved some sort of calculation with numbers given in the problem in order to get an answer.

The way the questions have been solved clearly shows that the child who is good at problem solving is actually good at the procedural aspect of problem solving. The other children who are not good at problem solving struggled with the conversion of mixed fraction to improper and doing some multiplication to get an answer showing that even the procedural aspect of such children is weak. Analysis of the answer sheets revealed that the students had problem dealing with fractions and 21.6% children of grade group D giving no answer may be attributed to it. A student in his reason for not responding to the problem stated “I do not understand 2□.” A common reason given by children for not giving a response to the problems was that they did not understand the question or they did not know which mathematical operation was to be applied in the given problem.

Children from all grade groups tried to get an expected answer by some or the other mathematical calculation without understanding the problem contextually. From the analysis it can be said that higher percentage of low achievers (grade C and D) as compared to high achievers (grade A+ and A) give other answers which are wrong answers according to classroom assessment. High achievers (grade A+ and A) give more expected answers (right answer required in the classroom) than low achievers. This clearly shows that low achievers try and solve problems by mechanical application of mathematical operations trying to '*do something*' with numbers given in the problem without understanding it while the high achievers know which mathematical operation to apply to solve the problem and get a legitimate correct answer without considering it contextually. It is clear that students' approach, that is, being procedural or contextual is not related to whether they are good at problem solving or not.

Conclusion

From the above analysis it can be concluded that children's approach to the realistic word problems is highly procedural. The high achievers (grade A+ and A), average achievers (grade B) and the low achievers (grade C and D); all have a procedural approach towards problem solving. The low achievers usually give other answers (wrong answers) due to their lack of competence in carrying out numerical calculations and choosing correct mathematical operations. It is evident that their dilemma is focussed on which arithmetic operation to apply to get a legitimate correct answer rather than on understanding the problem, like a girl was asked how she arrived at the answer she said, *"If the answer is wrong then I don't know whether I have to divide or multiply."* It can be concluded that children who participated in the study followed the implicit rules that there is only one exact numerical correct answer to every word problem and that the answer must be obtained by performing basic arithmetical operations on all numbers stated in the problem. The children who gave no response to the problems stated that they did not understand what to do in the problem, that is, which arithmetic operation to use.

The school teaching, which is highly procedural, socialises a child to learn the implicit rules of problem solving in classroom. Farida Abdulla Khan (1999) studied and compared two groups of vendors and a group of school children for differences on their knowledge of number systems and their competence and understanding of a set of mathematical word problems and found that vendors had a better understanding of the mathematical principles and a better range of strategies than the *schoolchildren, who were constrained by a narrow application of school-learned routines and algorithms.*

Those children who attain high grades are high on the procedural aspects of problem solving and low on the realistic considerations while those who attain low grades are unable to understand even the procedural aspects leave alone the contextual aspect. Both these groups are equally unable to employ their everyday knowledge to solve the problems and show strict adherence to the implicit rules learnt in the classroom. Students' school grades in problem solving in mathematics are not related to their understanding of realistic consideration of the word problem. Thus it is clear that in this context the stereotyped classroom teaching did not foster 'mathematisation' of thought in children and the gap between classroom mathematics and mathematics outside school still remained.

The Key Issues and Concerns of NCF-2005 for Science and Mathematics Re-emphasised for their Easy Implementation, to the question 'how do you visualise achieving the higher aim of mathematics in our education?', states that it is possible by developing the child's capability for logical and analytical thinking, nurturing a confident attitude to problem solving, and an ability to decide which mathematical tools are appropriate in which context and to apply them accordingly. Thus the responsibility falls on the teachers to nurture an environment that develops these abilities in the children. They should guide knowledge, but allow the students to experiment, manipulate objects, ask questions and try things that don't work, allowing them to integrate new experiences and interpretations to construct their own personal meaning. They should allow multiplicity of approaches to a problem discouraging a single method as the right method and a single answer as the only correct answer, liberating mathematics from the stereotype of getting a right answer found by applying the one taught algorithm. If children's classroom experiences are to be organised in a manner that permits them to construct knowledge, then our teacher training programmes should focus on empowering teachers to use new methods for teaching and learning of mathematics and break away from the set stereotype, so that the goal of 'mathematisation of thought' is realised.

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