Making Learning of Mathematics Developmentally Appropriate

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Abstract

'Good' performance in mathematics at elementary level has always been a concern for learners, as it indicates the individual's ability of alacrity, accuracy and brevity in thinking and estimation. It, moreover, projects an individual's capacity of reasoning, critical and analytical thinking. Society also places high importance in performing successfully in this subject as it is considered to be the catalyst for children to gain 'good' employment in their adult lives. But for majority of children, learning of mathematics tantamount to 'swallowing' of mathematics and 'memorization of formulas'. Therefore, learning of this subject has been observed to draw considerable amount of fear and anxiety accompanied with a feeling of incompetence, which over and over again estrange children from school and play an important role in their non-participation and disinterest in school activities, irregular attendance and/or drop-out. Besides, RTE-2009 makes it a necessity to provide every child with 'good quality' elementary education and hence it becomes an urgent need to ensure that children learn mathematics in a constructive and conducive environment, wherein learning of mathematics is more meaningful and learner centric (NCF-2005). In this backdrop, attempt was made to explore a few psychological dimensions involved in the learning of mathematics during elementary stage, one of which was delving into the concern of appropriateness of the content in terms of their comprehensibility vis-à-vis learners' cognitive development and age/grades. The exploration involved deliberations with subject experts, mathematics educators and psychologists, focus group discussion with practising teachers and learners to congregate their perspectives, analysing of present mathematics textbooks (Class I to VIII) and reviewing of literature. Findings are discussed with implications for subject teachers, teacher educators and mathematics curriculum framers and textbook writers, in the Indian context.

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Introduction

In the recent years extending of psychological principles in learning of specific school subjects has gathered momentum. Learning of all school subjects at the elementary level is crucial for building the base of any subject, during these years of education. All the same, importance of understanding psychological dimensions in learning of mathematics takes priority, because mastery over this subject manifests an individual's ability of speed, accuracy, neatness, brevity and estimation. It also projects an individual's capacity of critical, logical and analytical thinking and reasoning. These faculties help individuals in analyzing and solving not only day-to-day problems but also problems related to social and economic development of the society. Probably because of this, society also places high importance in performing successfully in mathematics. Learning of this subject is considered as the catalyst for children to gain 'good' employment in their adult lives. The better one's ability in learning and performing in mathematics, the higher her/his chances are of getting into engineering, medicine, public administration and management professions.

However, learning of mathematics besides that of science and languages has been observed to draw considerable amount of fear, anxiety and feeling of incompetence in school children. This fear, anxiety and feeling of incompetence often estrange the children from school and play an important role in their non-participation and disinterest in school activities, that further leads to irregular attendance and/drop-out. One may reason that in a classroom there may be a few children who enjoy learning the subject and 'do mathematics', but for majority it is 'swallowing' of mathematics and 'memorization of formulas'. In order to develop 'mathematisation of thinking' National Curriculum Framework (NCF, 2005) thus geared towards making learning of mathematics more meaningful and learner-centric.

Teaching and learning of mathematics is a complex activity and many factors determine the success of this activity. Mathematics is not only hierarchical in nature but also highly abstract. It is concerned with ideas rather than objects; involves manipulation of symbols rather than manipulation of objects. Nature and quality of instructional material, presentation of content, learning environment, motivation of the students are all important factors involved in learning of the subject. Also teaching of this subject is concerned with the computational know-how of the subject, selection of appropriate mathematical content as well as its appropriate communication, only then it leads to adequate understanding and application of the concepts. Due to the inherent nature of the subject, dependency of learners on teachers for optimum learning is heightened. Thus, factors as mentioned above, probably, accelerate the difficulty in learning and comprehending of the mathematical concepts and continue to remain complicated for children at all levels, be it primary, middle or secondary.

In the context of Indian classrooms this gains further propensity. Large number of students in classrooms, low teacher student ratio (in fact there are also multigrade classrooms with single or two teachers) and a diverse socio-cultural milieu are to name some of the challenges. Thus, those who wish to make the process of learning mathematics developmentally appropriate, in order to make it an enjoyable experience rather than one to be endured, are often faced with difficulties either due to large classrooms and/or divergent language and other social-cultural background, besides divergent mathematical ability of her/his students.

The apparent disconnect, lack of understanding and relating of the concepts and contents taught in classroom gains propensity with every passing grade and becomes noteworthy as children move from primary classes to completing of elementary stage. It is pertinent to notice that as the children transit from primary to middle school, they also are in zenith of their physical and psychological development. The specific Classes of V, VI, VII and VIII which mark the transition and also completion of elementary schooling, correspond with not just rapid physical and cognitive development but are also marked by several crisis and search for their resolutions (in form of quest of one's identity) which go a long way in the self development of the children as individuals. Therefore, on one hand undergoing the tribulations of transition from childhood to adulthood and on the other the demand of mastery over a subject that is not only critical and complex in nature, but also depend upon factors that are beyond the control of children, probably make learning of this subject difficult, leading to loss of interest and motivation.

Furthermore, the societal norms that point towards optimal mastery on this subject as an indicator of being 'intelligent and good' student, plays overwhelming role in the development of the self. Performing/non-performing in the subject puts a social tag on the individual at home, school and also among peers with regard to an individuals' present ability and future performance in adult life. All these connotations further act as a barrier in opening up to the complex nature of the subject.

However, mathematics has a fundamental role to play in enabling cultural, social and technological advances as well as empowering individuals as critical citizens in contemporary society and for the future. Number, space and measurement, chance and data are common aspects of most peoples' mathematical experience in everyday personal, academic and occupational situations. Equally important are the essential role that mathematical structure and working mathematically play in peoples' understanding of the natural and human worlds.

Mathematics learning, hence as seen, carries with it certain burdens, which act as blocks in children's ability and motivation to learn the subject. These blocks, as mentioned above are firstly the very nature of the subject, which is hierarchical. In other words, it is built layer upon layer of previous learning – thus making mastery of previous layer(s) a necessity to progress and learn the forthcoming layer of concepts and procedures. Second block is the societal expectations and pressure of 'high' performance in the subject. Since 'successful' performance in the subject projects an individual's capacity of critical, logical and analytical thinking along with the ability of reasoning (faculties considered essential in solving economic and social concerns) and hence increasing ones probability of getting into coveted professions. This demand of the society to perform in this subject (else be labelled as 'good for nothing') coupled with the inherent 'hierarchical' nature of the subject, is probably what causes in a learner fear of performance, fear of ridicule, fear of failure and thus becomes the third and final obstruction in learning of the subject. Unfortunately, failure as well as perceived failure in performing in this subject, is among the frequent causes that compels many students (through elementary years) to give up not just learning of mathematics but also attending school. These factors make it a priority to understand some psychological dimensions that are involved in learning of mathematics, one of them being the concern of appropriateness of the content in terms of their comprehensibility vis-à-vis learners' cognitive development and age/ grades.

Objective of the study: The present study made an attempt to explore the concern of appropriateness of the content in terms of their comprehensibility vis-á-vis learners' cognitive development and age/grades, during elementary stage.

Methodology: In order to accomplish this objective the study adopted strategies of deliberations (both formal and informal) with subject experts, mathematics educators and psychologists, reviewed literature, held focus group discussion with practising teachers and learners and also analysed present mathematics textbooks of the NCERT.

Findings: Attempt was made to understand the issue from theoretical underpinnings and literature survey as well as the viewpoints of learners and teachers on the issue. It is pertinent to mention at this juncture that the analysis is not purely based on empirical data, though as mentioned earlier, learners and teachers were interacted with and their viewpoints were collected through focus group discussions.

A crucial factor influencing learning is the maturational and experiential readiness of children. It is particularly important in the context of learning mathematics which requires mastery of certain pre-requisite concepts, skills and mathematical vocabulary learning. 'Curriculum load' often surfaces as one of the causes that probably make learning of mathematics difficult for most children. 'Curriculum load' can be due to an overload of information (i.e., content) and/or the 'load of noncomprehension' (Kaul et al. 1995). According to Kaul et al (1995) the 'load of non-comprehension' can be attributed to a large extent to the mismatch of developmental and academic priorities in framing of curriculum and also the classroom prac tice.

Theoretical standpoint and literature review: We take a look at some of the theoretical perspectives of child development and deliberate on them along with relevant research findings.

Piaget's cognitive development theory: According to Piaget, concrete operational period (roughly from 7–11 years) is a period in which children acquire certain logical structures that allow them to perform various mental operations, which are internalized actions that can be revised. In fact, he sometimes combined ages 2–11 years and labelled it as "preparation for and achievement of concrete operations" (Miller, 1993). Piaget believed that during the concrete operational period children developed and mastered various mental operations such as reversibility, compensation, class inclusion, relation, temporal-spatial representation, etc. However, it is important to note that firstly these concepts/operations do not develop at the same time (Miller, 1993). It is well known that comprehending 'conservation of weight', often do not happen until nearing the end of concrete operations (thus approximately around the age of 11–12 years). Secondly, each cognitive acquisition develops over a period of time, it gradually moves from a transitory stage to being strengthened, stabilized and generalised to variety of situations (in other words attain maturity for application of the concept in variety of situations). So the operation of reversibility may begin at approx 7-8 years (when it is in a transitory stage) and then gradually progress over the years to get strengthened and stabilized.

Thus, during concrete operations (roughly corresponding to our primary schooling) children are capable of understanding based on representations that are internalized and organized. Their thoughts get decentralised, dynamic and reversible, all reflecting a capability of a logical system in thought process. However, children still need "concrete" representations of objects/ mental presentations. They can deal with "what is" rather than "what could be" – i.e. they can deal with the 'actual' rather than 'possibilities'. Thus, concepts and skills that require them to shift from hands-on experiences to planning, inferring and deducting is difficult as they are still comfortable with focusing on a single operation/thought only (particularly, in the early primary years of Classes II-III).

It is important to note here that in these classes we wish children to 'guess', 'estimate', calculate multiple digits (addition, subtraction, multiplication, division) and even do reverse calculations, understand problems through stories and solve them; including those that have multiple operations (for example addition followed by subtraction, etc. through a story). Also in Class II we have stories where children are asked to 'guess' why a sack of salt when soaked in water becomes lighter, while a sack of cotton becomes heavier or the example of balancing the 'see-saw'.

Difficulty with word problems: Often it is perceived that word problems create confusion in the young learners as they demand high levels of comprehension ability (thus probably moving the focus away from numerical to verbal ability). Word problems are also seen as those that require an ability to infer, deduct and also involves multiple operations. Word problems in Indian mathematics textbooks, at present, consist of 50% in the 'combine' category and another 35% in 'change' category (Menon, 2007). It was further observed that there is a need to consider such problems from the perspective of 'mathematisation' of the real world. At present Menon (2007) pointed out that, such problems are not being prepared to model real life situations. She further suggested that there is a need for major restructuring of the mode in which activities (with/without concrete materials) are proposed in the mathematics classroom, so that children get the opportunity to solve word problems related to all four categories of 'change', 'compare', 'combine' and 'equalise' (Carpenter et al, 1983).

It is necessary to highlight here that, National Assessment of Educational Progress (NAEP) in USA have revealed over the years that majority of students in that country's context have mastered various mathematical operations after roughly a year or two of the content load being covered in the curriculum (Crown, 1990). However, even in such assessments it was found that a large percentage of 4th-5th graders had difficulty with regrouping (borrowing), particularly if it was given in the form of an instruction, such as "subtract 237 from 504" (Crown, 1990, pg. 509).

Neurological development of the brain that support Piaget's stages of cognitive development: Epstein (1986, 1990) had demonstrated that there are 'spurts' and 'plateaux' in brain growth, which tend to match with Piagetian stages of development. Thus the brain growth spurt at the age of 5-6 years is 'genetically' oriented for the development of concrete operations, while 10-11 years for formal operational functioning. Neurologically what occurs at these stages is an increase in inter-neuronal dendritic growth, thus preparing for more complex wiring-up in relation to experiences that are to come in future. This then demands that the child be exposed to 'appropriate' experiences, so as to utilize the new nerve fiber growth and in turn facilitate new neuronal connections. Thus, according to Kaul et al. (1995) these brain growth 'spurts' periods are those during which interventional experiences (in the form of learning new concepts and operations) have maximum effect. Infact, cognitively oriented interventions at pre-school stage have been found to have significant effect on academic achievement all the way through 8th grade (Clement et al, 1987). Kaul et al. (1995) further proved this in mathematical learning in the Indian context.

Information – processing theory of cognitive development: According to this theoretical approach, young children (upto 10 years of age) are capable of using rehearsal to aid memory, but they cannot spontaneously produce the strategy. They also lack in knowing when, where and how to use make use of these strategies, effectively. Information-processing theorists believe that by the time children enter their teenage they become more capable of picking a strategy that fits a particular task and carry out that strategy spontaneously, quickly and efficiently.

Researches in cognitive science also point out that in the first few years of elementary schooling (approximately upto 9 – 10 years) children have difficulty in the "inversion principle' (i.e., the idea that adding and subtracting the same number leaves the original quality unchanged). According to Siegler (2003) not until 11 years of age (i.e., roughly around sixth grade) do most children demonstrate an understanding of inversion principle and solve problems related to it, both quickly and efficiently. Another concept that takes time of children to understand is "mathematical equality". According to cognitive science researches, majority of 3rd and 4th grade children do not understand the meaning of 'equal' sign and thus commit errors (Alibali and Goldin-Meadow, 1993; Goldin-Meadow, Alibali and Church, 1993). Warren (2006) indeed found in a longitudinal study for three years on children from Grade 3 onwards that most children had limited understanding of 'equal' as a symbol of quantitative sameness, as well as had difficulty in comprehending 'more' and 'less'. She continued to reveal that over the three years period this difficulty in understanding 'equal', 'more' and 'less' showed no significant change. Cognitive scientists have also pointed out that multi-digit addition and subtraction is difficult and that children spend 'several years' learning multi-digit arithmetic. It is known that to learn this, children need to first learn the 'carrying procedure' for addition and 'borrowing procedure' for subtraction. To understand these procedures one needs to understand the place value concept (i.e., each position in a multi-digit number represent a successively higher power of ten) and also that multi-digit number can be represented in different ways. Many children, because of their difficulty in understanding the place value, adopt faulty procedures to solve the problems (Fuson and Briars, 1990) and thus create confusion when based on these operations further complex operations are required to be done in the higher levels of elementary mathematics. Fuson et al. (1997) in fact were of the view that teaching of standard algorithm for number operations need to be withheld, till children are capable of constructing such procedures. Researchers have shown that a combination of conceptual understanding of place value system and flexible procedures for operations with teaching standard algorithm helped in enhancing students' understanding (Ma, 1999). That place value is a difficult concept to understand and equally difficult is its procedural knowledge for primary grade children from second to fifth grade has been revealed in several researches (Kouba, Carpenter and Swafford, 1989; Ross, 1986; Fuson and Briars, 1990; Stevenson and Stigler, 1992). There is felt need to reflect upon the difficulty of multi-digit operations and what alternative procedures based upon number concept can be developed.

A look at the present NCERT textbooks reveals many such instances, in Classes II, III and IV where children are asked to do multidigit along with multiple operations, through word problems. For instance, children are required to first do multidigit addition and then decide whether the character in the word problem can carry all the stuff, thus implying that the child finally has to do a subtraction (multidigit) to get the reply. Similarly calculate the cost of *x* bricks from a range of differently priced bricks per thousand pieces, calculate the distance of *a* from *b*, when information provided is that *a* is 24 kms away and *b* is 46 kms away in opposite direction.

Difficulties with number operations by most children also makes it relevant to look backward to the learners' initial exposure to formal as well as informal learning of numbers. Mukherjee (2001) put forward that inadequacy in taking cognizance of children's intuitive and informally learned preschool mathematics knowledge, is probably a cause of creating confusion and fear in learning of mathematics at later stages of schooling.

Socio-cultural theory of cognitive development: While the above perspective delved into finding developmental appropriateness of learning the contents of the subject, there is another perspective which focuses on the teaching-learning process. According to the Vygotskian principle, children can learn complex operations if facilitated and surrounded by experts (in the form of parents, teachers, elders). There has been innumerable researches which indeed reveal that with a facilitative teacher, children's mathematical learning can be accelerated (Blanton and Kaput 2005; Mousoulides, Pittalis, Christou, 2006; Wilson, 2008; Burton, 1991).

Ma (1999) in her book *Knowing and Teaching Elementary Mathematics* had emphasised that in order to encourage conceptual understanding of mathematics amongst children at their elementary level, it is essential that teachers at this level have a conceptual understanding of mathematics. Interestingly, Soto-Johnson et al. (2007) attempted to implement the learnings of Ma's work in preparing pre-service elementary mathematics teachers and found that those who adopted the strategies were indeed more effective as mathematics teachers.

Researchers (Sheffield, 2009; Wilson, 2008; Burton, 1991) have indeed highlighted that when children are made to understand mathematical concepts and the skills of problem solving accompanied with a 'why', encouraged to verbalise mathematical reasoning through active reading, listening and writing and make effective use of peer group collaboration (particularly in a heterogeneous class) have led to promote effective learning of mathematics at all levels (primary as well as upper primary). Peer collaboration have also proved to be an important appendage to mathematical instruction (Gupta, 2008; Mousoulides et al., 2006; Turnuklu and Yesildere, 2007; Anderson and Kim, 2003).

It is pertinent to note here that primary teachers are required to teach all subjects to their students from Class I– V, including mathematics. Many of these teachers have not studied mathematics beyond secondary level and more importantly they do not have much interest in the subject (their mastery has been other areas like language, social science, etc.). Therefore, for such teachers to build up conceptual understanding of children in Classes II to V (no matter how simple they are) is not only cumbersome but also a 'burden', because they themselves probably do not have their concepts cleared. A Sarvodaya Vidyalaya (Delhi) teacher expressing her and her colleagues difficulty during the focus group discussion asked: "How much do you want us to do? I am a teacher with chemistry background. For me teaching SST is a nightmare." Similar is the case with someone whose background is in Hindi or SST. For them teaching mathematics is a 'nightmare'.

But for using all these and more such pedagogical skills it is equally important that the teachers have adequate content knowledge as well. As has been seen in researches by Anderson and Kim (2003), Turnuklu and Yesildere (2007) and others, teachers (particularly primary and upper primary) are required to have a sound base of mathematical pedagogical content knowledge to become effective facilitators in learning of mathematics. Blanton and Kaput (2005) in their work proved that when trained appropriately a primary teacher could effectively integrate algebraic reasoning into one's daily classroom instructions in planned and spontaneous ways that led to algebraic reasoning development among her 3rd grade students.

Distinction between performance and competence: It is also relevant to note the distinction between competence and performance. Children of about 4 years and younger have basic competences but may not have, yet, perfected their skills of performing in actual counting and other similar mathematical activities. Hence while arguing about children's ability to learn numbers one has to be careful of two factors, i.e. the awareness of 'how to count' principle and 'number conservation'. While the former may be possible (and actual experiences indeed reveal) that children below 6 years are very much capable, the latter, i.e. conservation of number is not possible until children reach 6 years and beyond.

Learners' viewpoint: We take a look at some of the viewpoints shared by learners during focus group discussion.

Difficulty in subtraction and division at primary level: At primary level most children reported to have faced difficulty in subtraction and division followed by multiplication (including the ones who were high achievers). In subtraction they had difficulty when calculation involved deducting larger digit from a smaller digit (for example: 37–19). The difficulty was heightened when there were multiple digits (like: 617–139). This implies that their 'borrow'/carry over concept was not clear. Similarly, in multiplication of multiple digits by multiple digits, children had difficulty.

Learning of the number tables was a challenge to many of the children. In fact, most average achievers continued to have this difficulty even in their upper primary classes and instead of recollecting the table from memory, they made simple calculations with pen and paper, to solve complex operations (which caused them to take more time!). As for multiplication of multiple digits by multiple digits (such as 3172513) children said they managed to do either the multiplication but made mistake in addition or made error in multiplication. It was felt that here too the 'carry over' part was leading to confusion and that children could concentrate on only one operation and lost concentration and attention when another operation was demanded to complete the problem sum. As for division, having difficulty in subtraction was itself a cause of difficulty in division, but along with it was the difficulty in dealing with 'putting the decimal' (*Kaha 'bindu' lagaye samajh mein hi nahi ata – a Class VII student's reflection*).

When asked how did the learners' manage to get over with their above, mentioned difficulties their responses varied, such as taking guidance from elder sibling, class teachers explaining them 'how it is to be done', private tutors 'showing' them the process and making them practise, thinking on their own and finding out strategies and then practising accordingly (this being a trend only amongst the high achievers). However, it was also reported that there were some calculations which they continued to find difficult, but did not share it with their maths teacher (for fear of being ridiculed in front of the entire class – "that so simple a thing s/he did not know") and thus worked on the difficulty, silently!

Similarly in upper primary, majority of children (irrespective of their achievement level) had difficulty with integers, learning different types of angles, fractions and decimals and operations related to them, and understanding algebraic operations. Learners also had difficulty in formulating equations out of word problems (related to speed/age/distance problems).

Viewpoint of Teachers: The primary teachers (across government schools, Kendriya Vidyalayas and private schools) were of the view that majority of children had difficulty in addition and subtraction that involved 'borrow and carry', long multiplication and division and solving story problems.

In upper primary level, a teacher from Kendriya Vidyalaya reported that though she spent considerable amount of time introducing and explaining 'variables' to her children in Class VI (as a part of introducing algebra and realized that only a handful were able to understand her), she had to repeat the same exercises and examples to the same group of children when they moved to Class VII (where interestingly most of the students could easily understand them).

Besides it was observed that difficulties that were voiced by the students were same as the ones highlighted by teachers as areas that they found most of their students having difficulty. They had to, often, take help of stories/daily life situations/hands down experience of understanding formulae with paper and cuttings (and other innovative techniques) to introduce as well as clarify the doubts of learners on the concepts of integers, fractions and decimals, algebraic operations, etc. Also they had to spend considerable amount of time in clarifying doubts of children in mathematical operations learnt in their primary classes.

Thus, here again the question arises as to whether the areas (where children and teachers have voiced difficulty) are appropriately matching to the learners' cognitive development? Why is it that most children have difficulty with the same procedural skills and that too their difficulties are of similar kind?

Expert's Viewpoint: Experts were of the view that mathematical understanding and memory develops in a sequential manner. Furthermore, it was essential that children by the end of primary level have understood and learnt the four fundamental operations along with understanding of shapes, so as to make mathematical learning at secondary stage comfortable. The above opinion came up as it was observed that in majority of cases children did not have the basic understanding of number concept, place value concept and geometrical understanding when they began mathematical learning at upper primary stage. This probably was a contributing factor of feeling the 'big jump' in learning of the subject as children moved from primary to upper primary (middle) schooling.

Discussion and Implications: The above explorations throw light on differing theoretical perspectives and viewpoints of learners, teachers and experts, which are discussed and some implications are made.

Piagetian viewpoint and relevant literature review have revealed that children in the years of primary schooling (i.e., roughly corresponding to the concrete operational period) can deal with the 'actual' rather than 'possibilities', 'with what is' rather than 'what could be'. In other words, those concepts and their relevant skills that require the children in the years of concrete operational phase to shift from 'hands on' experience to planning, inferring and deducting are difficult, as they are still comfortable with focusing on a single operation/thought only. As they progress with age and cognitive development they gradually internalize, comprehend, strengthen and stabilize the 'hands on' experiences and develop the skill of inferring, deducting, etc. Research evidences in the field of neurological development also have supported that there are spurts of growth in the brain during the age of 5-6 years and again around 10-11 years, when inter-neuronal dendritic growth increases manifolds, thus making these phases of developing years very crucial to receiving and learning 'appropriate' experiences.

Furthermore, investigations by cognitive scientists have pointed out that initial years of elementary schooling have proved to be a difficult phase for children to comprehend "inversion principle". It has also been observed that children take time to understand "mathematical equality" and also in understanding 'more' and 'less'. Another revelation, which is of much concern, is that children spend many years in learning multi-digit arithmetic. Researchers in the field of cognitive science have shown that not only children in primary years have difficulty with multi-digit arithmetic; they also adopt faulty procedures to solve problems. Due to this difficulty with 'place value concept' and end up creating more confusions and complications, when based on these operations complex operations are required to be done in higher levels of elementary mathematics. Several researches (Kouba et al., 1989; Fuson and Briars, 1990; Stevenson and Stigler, 1992) have revealed that place value, indeed, is a difficult concept and equally difficult is comprehension of its procedural knowledge by primary grade children. Learners during interaction with them had also voiced similar concerns of difficulty in subtraction, multiplication and division of 'big' numbers. Teachers of upper primary level had shared that often they had to revise primary level learnings (as most children were not clear) before beginning with more complex operations.

This has implications for not only curriculum planners and textbook writers, but also for researchers, to delve into finding the 'appropriate age' and cognitive development when learning of multi-digit arithmetic is feasible. Also the need is to investigate into alternative approaches of learning and teaching multi-digit operations.

Moving ahead with the learning of the subject, much responsibility has been put on the teachers who communicate and help learners comprehend its various concepts and procedures. The socio-cultural theory of cognitive development emphasizes (with ample research evidences, as already mentioned in the findings) that children's mathematical learning can be much accelerated with the help of a facilitative expert (including a teacher). However, this demands that the teachers are given adequate training and exposure both in the subject as well as in instructive (general as well as subject-specific) skills. Discussions with primary teachers, in the present study, have revealed that the primary teachers often feel themselves inadequate in dealing with students queries (because they have specialization in other subjects like SST or language, refer to box on page 109-110). As a result of their inadequacy, the students depend upon faulty procedures and their concepts remain unclear, even as they move to upper primary classes.

Therefore, another implication for research, development and training is the need of building the capacity of teachers (at all levels, including the primary teachers) in mathematics subject-specific pedagogy, particularly in those areas that are considered as 'hard spots' at primary and upper primary levels.

However, with practising teachers sharing the same examples while introducing and explaining variables to students of 7th grade, were more meaningful for them than a year before when they were in 6th grade, one still wonders about the appropriateness of the content (in terms of comprehensibility) vis-á-vis learners' cognitive development.

It was interesting to observe that there were some gaps in the continuity of contents in the upper primary classes. In geometry there was apparent discontinuity which probably made the learning of it cumbersome for students. For instance, though in Class VIII students were expected to handle construction of angles, triangles, quadrilaterals as well as special quadrilaterals, yet in the previous class (i.e. Class VII) there was very little exposure in these areas and hence most of the students were not prepared to handle such construction, even to make use of the compass. Another example of discontinuity in content was also observed in the learning of 'congruency of triangles'. Though this is introduced in Class VII it is dealt in much detail in Class IX, where students are expected to apply the concept while working out geometry. With no follow up in the intervening class (i.e Class VIII) students tend to forget the learnings of Class VII (due to lack of practice) by the time they reach Class IX. However, the remedy may not just be in reducing the content load or questioning the age appropriateness of the content, but probably also in creating an empathetic, understanding, motivating environment of mathematical learning. This is because for the young and inquisitive minds, a four walled classroom is too small a canvas and the proceedings of the class, if not meaningful are often felt 'boring'.[1]

For instance, it was observed that with geometrical patterns and shapes the focus was more on ensuring that the learners answered to 'what' and 'which' questions rather than the 'why'. Even in the primary classes where shapes and patterns have been dealt, teachers found it irrelevant and there was not much emphasis on the need to help learners recognize and understand various shapes and their properties. Thus, developing of spatial reasoning abilities in the children, which is the 'main purpose' of school geometry (*Source Book on Assessment* for Classes 1–V, Mathematics, NCERT, 2008) was probably neglected during primary years of mathematical learning. Probably this is the reason why children in their upper primary classes have difficulty in understanding the properties of geometrical shapes and perform relevant operations. Menon (2009) indeed argued that with appropriate instructions and scaffolding children even in primary grades were capable of developing an understanding of angle concept. van Hiele (1984) cited that difficulty and failure of middle school geometry was primarily because of the discrepancy between the teachers' use of language of instruction of a higher level than the students' level of geometrical thinking. According to the van Hiele theory there are 5 levels of geometrical thinking — visualization level, analysis level, ordering/informal deduction level, formal deduction level and rigour level (Menon, 2009), in which transition from one level to another is more (and strongly) dependent on instruction, rather than a spontaneous transition. Researches by Usiskin and Senk (1990), Human and Nel (1997/87), Clements et al. (1999) have further proved the validity of this theory.

Therefore, mathematical learnings need to be planned, so that learners of primary as well as middle school find the learnings relevant to their life and daily living, and not just an academic requirement.Probably, there is not only a need to relook at the developmental yardsticks in the light of learning of mathematical concepts and procedures but also the continuity and transition of the contents from one grade to another and develop strategies to ensure a more active role of the teacher in developing

[1] 'Some boys in the class sit in the last rows and chat. They often find solving of big problems as a 'boring' and 'meaningless', and sometimes in the pretext of games teacher calling, they leave the class, one by one' — Class VII-VIII students sharing of their mathematics class scenario.

and facilitating understanding and learning of mathematics, both at primary and upper primary level.

Cobb (1995) indeed pointed out that interaction between what learners bring with them when entering the classroom and what they encounter there, often control the learning. Teacher-student relationship in learning of mathematics is a crucial one, because more than in any other subject, here the process of learning is dependent on agreement, which is purely based on reasoning. Both the teacher and students are subject to the same mathematical rules. Hence an authoritarian hierarchy of the teacher may not be applicable for imparting learning of the mathematical concepts and relevant procedures. After all, students respect greater knowledge of the teacher and also expect their own understanding to be enlarged, through interaction that is founded on respect of each other's ability.

However, when students (irrespective of the age/grade) encounter such situations in learning of mathematics wherein they are expected to learn rules that seem 'meaningless' to them and also are anticipated to solve equations based on these 'meaningless' rules, they lose interest and find no motivation. To the students, rules probably become meaningless, because firstly they are unable to find a reason and secondly, when others too cannot provide a reason. Over the years when such experiences gather together, it leads to gradual acquiring of lack of enthusiasm for the subject and sometimes a feeling of repulsion. This is of particular concern at elementary level, as lack of adequate stimulation in the years of rapid cognitive development may prove to be detrimental for their future development in the areas of numerical, spatial reasoning, estimation, brevity of thought, etc. all of which are expected to be nurtured and expanded through the process of mathematical learning.

Mathematical learning and performance involves cultural, social and cognitive phenomena which cannot be separated, and hence need to be understood in relation with each other. Subramanian (2003) opined that in case of children developing the concept of number (as well as in other domains) constructing knowledge may involve coordinating an artificial, culturally developed symbolic system with an intuitive or innate base of basic concepts. Several social theorists (Zeverbergen, 2001; Dowling, 1998) have also proposed that learning of mathematics demands establishing links between the performance of students to their social and cultural backgrounds, besides extending the views of socio-cultural theorists which is based on the work of Vygotsky (1978). Besides, Dodge and Bichart (2001) rightfully pointed that children in early grades of schooling need appropriate challenges so that they can feel successful (as this age group corresponds to the stage of industry vs. inferiority of Erikson's psychosocial developmental stage). This has also been shared by learners "like mathematics because I can solve the problems. This gives satisfaction and also willingness to solve more problems (*maths is liye achha lagta hai kyunki jab problem solve kar lete hai toh bahut maaja aata hai, maan karta* *hai ki aur problems solve kare*)[2]." Children during this stage are actively involved in the learning process and are full of energy to get hands-on (concrete) experience. Therefore, an apt environment in the school is required that encourages numerical and spatial reasoning through use of concrete examples and hands-on experiences, even during the primary years. Having discussion on solving certain equation(s)/problems in the classroom/school may be given as an activity. This however, demands the subject teachers to be prepared in the skill of conducting a good group discussion based on mathematical knowledge.

In a classroom, two kinds of authority play a significant role – one that of a disciplinarian and the other is resultant of 'superior knowledge'. While the former may be a necessity to establish and maintain certain amount of order in the class, so as to bring all learners on a common platform of 'readiness' for learning, the other (i.e. authority through superior knowledge) is required to ensure a dynamic mathematical classroom environment of reasoning, debate and agreement, at all levels. This is of utmost importance to deal with the concern of anxiety as an impediment in mathematical learning. When a learner does not comprehend the proceedings of mathematics discourse, they feel over-anxious at the apparent failure to understand what their peers have (presumably) understood. In such a state, one tends to make greater efforts to comprehend – which actually diminishes the effectiveness of their efforts, thereby further increasing their anxiety. In the long term, repeated experiencing of such anxiety-ridden situations in mathematical learning, there probably comes a time when the learner begins each lesson with anxiety, fear of failure and ridicule (this was also put forth as a feeling by learners)[3]

This implies that the mathematics teacher (at all levels) instead of encouraging anxiety and fear for the subject, probably need to make strategies to reduce the already existing baggage of anxiety before commencing a new learning and continue doing so through the entire process of learning. S/he also needs to have in-depth knowledge and conceptual clarity on the subject, so as to not only control her/his class but also generate curiosity, interest and motivation to learn. In such an environment learners will not endure the learning of mathematics, but enjoy it. Also, keeping in mind that mathematics is a hierarchical subject, wherein mastery of learning at each level ensures the learning of a concept/skill at the next level, subject teachers (at all levels, including primary) have to be assisted in implementing appropriate content specific pedagogical knowledge. This is to ensure that the teachers make their learners' learning of various operations and procedures a meaningful experience, based on reasoning.

Children as they progress through the elementary years of schooling are also undergoing various developments within themselves as well as in their social interactions. If these experiences are brought into their learning of school subjects, particularly mathematics, it may make the learning not only

[2] 'Like to do mathematics because we feel a thrill when we are able to solve a problem and then we wish to solve more problems'

[3] A boy of Class VIII shared that he (and many of his friends) do not share their difficulty with their mathematics teacher for the fear of being ridiculed in front of the entire class – which happens to be a co-ed one, and so they continue with their difficulties silently.

interesting and motivating, but also will be perceived as relevant and useful by the learners. Examples and activities related to their daily activities, situations at home, neighbourhood, including school, etc. where the mathematical procedures and operations can be applied, may be included in the teaching learning process.

Finally, learning and performance in this subject, as mentioned earlier, is laden with social expectations. Each and every learner brings with her/him a part of their society's understanding and relevance of learning the subject. Besides they also bring with them the informal learnings of number, shape and representation through signs and symbols. Be it in the primary or upper primary level, the focus of classroom transaction probably need always to be in connecting with such informal learnings of their culture and community, so that learners are able to consolidate and assimilate and thus expand their own schema and in the process internalize and develop the underlying ability of mathematical reasoning. For this mathematics teachers have to be equipped with the know-how of connecting the formal mathematical learnings with her/his learners' informal learnings. Particularly, where classrooms predominantly have large number of students from diverse language and cultural backgrounds, the above is a challenge for teachers, for which they have to be prepared during their pre-service training and continued also at regular intervals as sessions during in-service training.

Mathematics as a subject carries much importance not only academically but also socially, therefore, mathematics classroom environment during the elementary years probably need to be shaped so as to be stimulating, enriching and rewarding for all learners and wherein mathematical learning is more activity based rather than predominantly a paper-pencil accomplishment.

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