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Deeper Learning through Constructivism – A Case Study with Primary Children on Number Concept

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Abstract

"Deeper Learning" simply refers to "process of learning for transfer," meaning it allows a child to take what's learned in one situation and apply it to another. To elaborate the definition of deeper learning further, let us recall the three domains of competence– cognitive, intrapersonal and interpersonal. Cognitive refers to reasoning and problem solving; intrapersonal refers to self-management, self-directedness and conscientiousness; and interpersonal refers to expressing ideas and communicating, and working with others.

These three broad competencies are related to each other. If deeper learning is the ultimate goal, can it be taught? To a certain degree, yes. On the same pitch, in constructivism, the central idea is that learning is an active process in which learners construct new ideas or concepts based upon their current and prior knowledge. In this article the author has experimented with 20 children of Class II on how constructivism and assessment for learning within formative assessment, can support to achieve deeper learning. The entire discussion and activities for this experiment were based on number concepts. The author has realised that if students are encouraged with number patterns within inductive reasoning, they build the basic concepts of numbers like place value, increasing and decreasing numbers etc. Children also developed the number sense which helped them to build up an attitude to work on operation of numbers. In fact, the assessment and remedial positive teaching was planned during the discussions and activities. Children's own mistakes have been used as a tool to construct the concept. The author has realised that constructivism where children construct knowledge on their own, and assessment for learning within the lesson plan affects all the three domains – cognitive, intrapersonal and interpersonal, leading towards deeper learning.

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1. Introduction

Mathematics is perhaps the most demanding subject in its need for in-depth subject knowledge, even at primary level. In early grades, it is important that teachers working with children have a genuine understanding of mathematical concepts and language as they guide children's thinking and motivate exploration of patterns, shape, spaces and problem-solving. The central theme to effective mathematical pedagogy in the early years is fostering children's natural interest in numeracy, problem solving, reasoning in pattern recognition, shapes and measures. There are two kinds of reasoning that feature prominently in doing mathematics. Inductive reasoning is the kind of thinking involved in recognising patterns, similarities and equivalences, and using these to predict further results and to formulate generalisations. Deductive reasoning is the formulation of a valid, logical argument to explain, demonstrate or convince others that a solution to a problem is correct, or that a mathematical theorem is proved beyond doubt. Brownell claimed that 'meaning in mathematics is to be sought in the structure, the organisation and the inner relationship of the subject itself'. The structures, organisation and relationships within mathematics are obviously visible in the form of patterns. As a subject, mathematics expresses itself through patterns. Mathematics is full of patterns and relationships. In fact, Mathematics is known as a subject of patterns. In queries about the attributes of things, children apply inductive reasoning to answer what is next? Not with a number but with a description. Basically, it may always be a nice tool to initiate mind action in an interesting way. In our observation, we realised that if a child appreciates the involved pattern through visual and written forms of expressions, she/he can connect language and mathematics to develop skills for thinking clearly, strategically, critically and creatively.

2. Recognising Pattern

In general, it is not easy to define what we mean by "pattern", even in Mathematics. One of the difficulties is that the word has different meanings. On the one hand, "pattern" can be used in relation to a particular arrangement of numbers, shapes, colours or sounds with no obvious regularity. Indeed, sometimes the arrangement might form a recognisable representation or picture. On the other hand, it might be required that the arrangement possesses some kind of clear regularity or repetition in some sequence. In the world of music, an attractive melody written in a predictable rhythm is an arrangement of sounds in a regular pattern, which is somehow more memorable and appealing than most random sequences of noises. When we involve or appeal to pattern in teaching mathematics, it is usually because we are trying to help children to extract deeper meaning or enjoyment, or both, from the experience or learning environment with which they are occupied, and perhaps also to facilitate memorisation. In all walks of life, it seems we are attracted to regularity, and often try to interpret situations by looking at, or even imposing, pattern. Gestalt's psychology embraces the view that it is a human quality that we constantly seek to "interpret incoming sensations and experiences as an organised whole and not as a collection of separate units of data" Orton suggests that it would be positively helpful to children if they are encouraged to perceive, comprehend and then use patterns whenever possible in Mathematics. Mathematicians and educationists have long been enthusiastic about the importance of patterns in mathematics. Sawyer claimed that "Mathematics is the classification and study of all possible patterns". Williams and Shuard suggested that "the search for order and pattern is one of the driving forces of all mathematical work with children". Biggs and Shaw wrote: "Mathematics can be thought of as a search for patterns and relationships". In fact, such views of mathematics have been perpetuated by repetition and reformulation. Keeping these experiences of Mathematicians at central focus, our primary task was to generate a pattern at the beginning of counting numbers itself. In accordance with the number grid, we started concrete representation of number grid by putting unit cubes one by one and after having ten unit cubes, it was replaced with a column. The expectation was that the concrete exercise will help children to realise a pattern in the development of numbers. The classroom process and discussion was to set the number grid in the 'concrete number modelling'. This was done by the children themselves. In terms of inductive reasoning, the objective was that children should be able to recognise simple patterns and relationships and make predictions about their understanding; investigate general statements; search for pattern in their results.

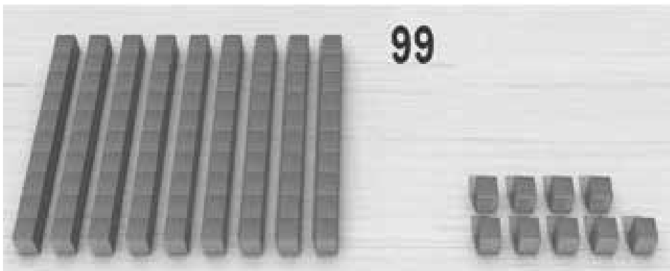

Figure 1. Number Modelling

3. The Classroom Practical

At the start of new session during the 2012 summers, we were with Class II children of Government Primary School, Muthiyani, Gautam Buddh Nagar, U.P., for a period of 3 months from July to September. We began by using several mathematical kit items like beads, colour counters, unit's cube, ten's rod (having 10 unit cubes) and hundred's plate (having 10 ten's rod or equivalently 100 unit cubes) among the children. There were many children who seemed to be highly interested to play/work with these mathematical apparatus, and it was almost inevitable that, while doing so, they would notice and create patterns. We started with basic counting and during the classroom we played with straws, cubes, wooden sticks etc. We focussed on making collections of particular items for certain numbers. For example, a child named Vikas was asked to collect 24 straws and Ajay was asked to collect 17 cubes etc. They were also putting the things either in random manner or just keeping all the things in pocket. Children often choose to work with other play materials like wooden sticks, pieces of stones along with these kit items. To provide opportunities for pattern-making, the next step was to ask them to arrange the items as they wanted. Sometimes the activity was conducted in groups so that children notice each other's patterns. For the arrangement activity, we could discuss different designs for arrangements. Arrangements in different designs were the first step to motivate towards the word "pattern". During the whole class discussion, we also provided several graphics to recognise patterns. Also, these graphic activities helped to develop pattern-related thinking among the children. In the beginning weeks, these children only occasionally tried to verbalise their pattern-related perceptions and intentions during such activities. For about three weeks, children's thinking about patterns was shown only by what they did. For example, Ritesh was putting different coloured straws in an order as 1, 2, 3 etc., and was saying that he was putting the things in a pattern. On the other hand, Vikas was putting the straws like.. 1 straw, 2 straws, 1 straw, 3 straws, 1 straw, 4 straws. He was not verbalising any pattern. Hari Om said, 'it appealed to him as something beautiful'. Our emphasis was that children explore and create patterns using kit and other items. This may be seen as a significant feature of successful early mathematical learning. They started taking patterns as something that should be in some systematic order as per their own understanding. Now our next point of action was to connect this pattern understanding of children with the development of numbers in the number grid.

4. Constructivism

We all agree that the central idea of constructivism in learning is active participation of learners where they construct new ideas or concepts based upon their current and prior knowledge. Behaviourist theories of transmitting knowledge to learners failed. As compared to our day-to-day experiences of classroom practices, it has become clear that when we teach or tell learners something, we cannot assume that they will make sense of it in the way we intend. Each learner internalises the knowledge and makes sense of it in an individual way. In the classroom we think that we have taught or covered the syllabus, while children's mistakes reveal that they have not understood the concept. To be most effective, we need to understand how children learn. Constructivism focuses attention on the children's learning rather than on the teacher's teaching. We can talk about a constructivist view of learning, but not constructivist teaching in isolation as this is a contradiction. Rather than thinking about perfect definitions or explanations of concepts and skills, the challenge for us is to create experiences that engage children and make mathematical learning meaningful, which can be applied or transferred to other situations. Constructivism, as a theory of learning, is more than simply 'learning by doing' or 'experiential learning'. Although practical activities may go some way towards helping children to build up knowledge, activity kits are not sufficient as they do not embody a concept. Children may manipulate kit items in the prescribed way but may not be learning, or they might not be able to transfer their knowledge to more formal representations or to other contexts. What is also needed is reflection on the activity. This might be individual reflection but will more often be promoted through discussion with the teacher or with peers. In practical terms, then, what does this mean for the mathematics classroom?

If the implication of constructivism is that there is more to teaching than just telling or trying to transmit knowledge to children, then how can teachers foster the development of mathematical knowledge? Carpenter and Lehrer adopting a constructivist model, identified five forms of mental activity that promote mathematical understanding. The teacher's role is to ensure that pupils engage in such mental activities such as:

- constructing relationships;
- extending and applying mathematical knowledge;
- reflecting about the experience;
- articulating what one knows;
- making mathematical knowledge one's own.

5. Pedagogical Interventions

Being motivated with this theory and to experience it practically, with the objective of recognising an interesting pattern in number grid, we used concrete materials. Children were encouraged to model the number grid by unit cubes and tens' rod. We tried to build the numbers by putting cubes one after other exactly in the same pattern of number grid. In-fact, the following 100 counting grid is written on the wall of most primary classrooms. We realised that it may provide a wealth of opportunities for children to observe and explore patterns in number.

1	11	21	31	41	51	61	71	81	91
2	12	22	32	42	52	62	72	82	92
3	13	23	33	43	53	63	73	83	93
4	14	24	34	44	54	64	74	84	94
5	15	25	35	45	55	65	75	85	95
6	16	26	36	46	56	66	76	86	96
7	17	27	37	47	57	67	77	87	97
8	18	28	38	48	58	68	78	88	98
9	19	29	39	49	59	69	79	89	99
10	20	30	40	50	60	70	80	90	100

Figure 2. 100 counting grid

During modelling of 1 to 100 number grid, it was expected that children will realise the pattern by themselves as per the requirement of constructivist approach of learning. In the beginning children were encouraged to model numbers 1 to 30. A discussion was held while replacing 10 unit cubes by one tens' rod. Children were convinced that 10 cubes and a tens' rod are representing the similar number. It was easily grapsed by children and they were replacing ten separate unit cubes by a tens' rod while modelling of number 10 after the number 9. To move further, after a few discussions on number grid table, children started the modelling. In fact few children were repeatedly observing the number grid table after every step of their construction. While modelling numbers 11 onwards, after having one tens rod and putting cube one by one, they realised that it is something like the previous work for numbers 1 to 10. The learning of pattern was reflected in modelling 20 after 19 (replacing ten cubes with a tens' rod) and 21 after 20. We discussed enough about replacing 10 unit cubes by a tens rod and 8 out of 20 children completed the step of 20 after 19 by themselves. A brief discussion was held for rest of the class. This is the point where children were expected to connect their pattern exposure to move forward with confidence. Out of 20, ten children could get the pattern comfortably. Rest needed support to recognise the pattern. Less than a week's practice with some mixed and different approaches like choosing any number on the number grid and making its model or vice versa, gave clarity of patterns involved in number grid. Children were ready to model the entire number grid of 1 to 100.

While motivating children to recognise pattern during the modelling, we realised that we were on the trak of the requirement of mental activity that could promote mathematical understanding. We found that while modelling of number 100 after 99, few enthusiastic children replaced entire unit cubes and ten's rod by the plate of hundred by themselves. They were reflecting at the moment, and this is their own mathematics, because they got an appreciation on the work which they had done themselves without taking help from us. The number pattern recognition activity through modelling was performed for one and half weeks. Children also worked on reverse approach i.e., by looking on the model while noting down the number. In fact, this was started by Ritesh when asked to check the work of Vikas. Vikas had tried to model the number 53 with units cube and tens rods. But while modelling he has taken 3 tens rod and 5 unit cubes. Ritesh's model had 3 tens rods, Vikas said that the number of this model would lie in the column of numbers having 3 at left in number grid. And on further questioning, he explored that it will be in fourth column. It made everyone look at the activity in both ways from numbers to model and vice-versa. Through modelling activity, they also started feeling about the size of the number. During discussion, one of them reflected that since in 73, we are using more kit materials as compared to 43, so the size of number 73 is bigger. Probably this was an indication of number sense among the children. We realised that once children had got the basic pattern of number system, they reached to a series of the following pattern in an interesting way.

5.1. Patterns in counting grids

The simplest pattern involves the numbers along a column, as they increase from top to bottom by 1, and decrease by 1 from down to up.

41
42
43
44
45
46
47
48
49
50

Figure 3. Column from the counting grid

The numbers in a row have the same unit digit and the tens digits increase by 1; thus the numbers increase by 10 from cell to bottom of a row, as shown in figure 4.

→ Increase by 10 ←									
7	17	27	37	47	57	67	77	87	97
→ Decrease by 10 ←									

Figure 4. Row from the counting grid

These are the two basic patterns resulting from the structure of the number system, base 10, and having ten numbers in each column. There are many others, a few of which are illustrated in figures 5 to 8 by shading numbers in the 100 grid which students realised themselves while performing skip counting activities. For example, for skip counting of 3, they verbalise the pattern starting from 3 and encircling every 3rd. Although the encircling/shading of the numbers produces interesting visual patterns, it is important that children consider the structure which has created the visual patterns. Each pattern is produced because of the relationship between the 3rd number. Although the encircling/shading of the numbers produces interesting visual patterns, it is important that children consider the structure which has created the visual pattern. Each pattern is produced because of the relationship between the 'skip counting' pattern. For example, the 'skip counting of 5' is shown in figure 5. Children were asked to explain why they think this pattern occurs and to predict with justification, whether it continues if the grid is extended with numbers 101 to 200 and beyond.

1	11	21	31	41	51	61	71	81	91
2	12	22	32	42	52	62	72	82	92
3	13	23	33	43	53	63	73	83	93
4	14	24	34	44	54	64	74	84	94
5	**15**	**25**	**35**	**45**	**55**	**65**	**75**	**85**	**95**
6	16	26	36	46	56	66	76	86	96
7	17	27	37	47	57	67	77	87	97
8	18	28	38	48	58	68	78	88	98
9	19	29	39	49	59	69	79	89	99
10	**20**	**30**	**40**	**50**	**60**	**70**	**80**	**90**	**100**

Figure 5. Counting of 5

1	11	21	31	41	51	61	71	**81**	91
2	12	22	32	42	52	62	**72**	82	92
3	13	23	33	43	53	**63**	73	83	93
4	14	24	34	44	**54**	64	74	84	94
5	15	25	35	**45**	55	65	75	85	95
6	16	26	**36**	46	56	66	76	86	96
7	17	**27**	37	47	57	67	77	87	97
8	**18**	28	38	48	58	68	78	88	98
9	19	29	39	49	59	69	79	89	**99**
10	20	30	40	50	60	70	80	**90**	100

Figure 6. Counting of 9

1	11	**21**	31	41	**51**	61	71	**81**	91
2	**12**	22	32	**42**	52	62	**72**	82	92
3	13	23	**33**	43	53	**63**	73	83	**93**
4	14	**24**	34	44	**54**	64	74	**84**	94
5	**15**	25	35	**45**	55	65	**75**	85	95
6	16	26	**36**	46	56	**66**	76	86	**96**
7	17	**27**	37	47	**57**	67	77	**87**	97
8	**18**	28	38	**48**	58	68	**78**	88	98
9	19	29	**39**	49	59	**69**	79	89	**99**
10	20	**30**	40	50	**60**	70	80	**90**	100

Figure 7. Counting of 3

1	11	21	31	41	51	61	71	81	91
2	12	22	32	42	52	62	72	82	92
3	13	23	33	43	53	63	73	83	93
4	14	24	34	44	54	64	74	84	94
5	15	25	35	45	55	65	75	85	95
6	16	26	36	46	56	66	76	86	96
7	17	27	37	47	57	67	77	87	97
8	18	28	38	48	58	68	78	88	98
9	19	29	39	49	59	69	79	89	99
10	20	30	40	50	60	70	80	90	100

Figure 8. Skip Counting of 4

Working with patterns on the counting grid that resulted during different 'skip counting' numbers is an essential prerequisite for searching the patterns in the multiplication tables. Many of these apparently are simple patterns helped to explore place value and its representation in symbols. Some of these facts are mentioned here.

1=1	10-1=9
10+1=11	100-1=99
100+10+1=1111	10000-1=9999
9+1=10	10=10
90+10	10×10=100
900+100	9000+1000=10000

Figure 9. Set up of Number Grid

Observing such patterns is only the beginning of children's understanding of what lies behind the patterns i.e., the principle of grouping by tens. After working in such environment for few weeks continuously, children are able to recognise a pattern and also to extend it, without understanding why should the pattern exist and the mathematical structure which brings about its existence. The importance of children talking about describing and giving their reasons for a pattern cannot be over-emphasised, as this encourages them to find a meaning for the pattern and consequently develops their understanding of mathematics.

In fact, this constructive way of learning mathematics based on number pattern reflects an attitude of deeper learning. Simply defined, "deeper learning" is the "process of learning for transfer," meaning it allows a student to take what's learned in one situation and apply it to another. (James Pellegrino. Pellegrino also said "You can use knowledge in ways that make it useful in new situations". To deconstruct the definition of deeper learning further it emphasised three domains of competence: cognitive, intrapersonal and interpersonal. Cognitive refers to reasoning and problem solving; intrapersonal refers to self-management, self-directedness, and conscientiousness; and interpersonal refers to expressing ideas and communicating, and working with others. These three broad competencies are related to each other". There is good evidence that shows learners can lead to success not only in education, but also in career and health. In fact, conscientiousness is most highly correlated with successful outcomes.

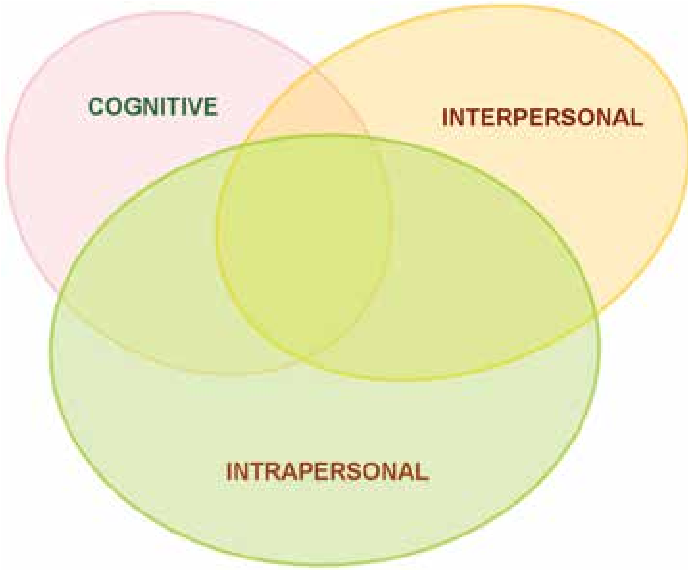

Figure 10. Inter-relation

If deeper learning is the ultimate goal, can it be taught? To a certain degree, yes. But for educators to engage in deeper learning with students, researchers say that they must begin with clear goals and let students know what is expected of them. They must provide multiple and different kinds of ideas and tasks. They must encourage questioning and discussion, challenge them and offer support and guidance. They must use carefully selected curriculum and use formative assessments to measure and support students' progress. Regarding assessment, Pellegrino said "Students can't learn in the absence of feedback". "It's not just assessing, but providing feedback that's actionable on the part of students". Since children realised patterns as a tool to handle the numbers, they could also think of the other tools. While checking each others' work they were also able to communicate mathematically.

In fact one incident was very interesting which also made us realise that constructivism promotes deeper learning. While discussing the central concept of place value, we asked one child to write the number name of "one hundred thirty two" on blackboard. A child named Hariom wrote it as 10032. We did not correct it. We asked him how you get 10 after 9. He replied, it is one more unit. When asked if you add one more unit to it? He thought a little bit and replied 11. This lead to same discussion on what happens from 99 to 100 and 100 to 101. Hariom realised on his own that there is some error in earlier response and corrected it as 132. While discussing, Hariom told that he tried to connect the number with unit cube, tens rod and hundred plate modelling of numbers and then could understand what was the error. An inspiring moment for us too. Constructivism works and it promotes the intrapersonal skills for deeper learning.

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