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Mathematics that We Know and Use

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Abstract

People, generally, have a wrong notion that mathematics is a difficult subject and believe that it is impossible to enjoy it. It is a fact that math is a part of everyday life. The understanding of mathematics comes very early and naturally in life. In other words, every one can enjoy, engage with, and relax with mathematics. Mathematics is much more — in fact, a lot more than numbers and working algebra formulas. These aspects sharpen the mathematical skills that one may already posses, just as speaking and writing is learned more by language skills. Many subjects and aspects are so deeply integrated with mathematics that it is hard to define it. Mostly in schools, one definition of mathematics prominently learnt is, "Mathematics is the study of quantities and relations through the use of symbols, numbers and rules". This article will help you to appreciate how mathematics is not only a part of you but of animals as well. Secondly, you will see how mathematical problems can be solved faster and with ease. You will also see how to skip the steps entirely and still find accurate or workable solutions to mathematical problems, perhaps without even using a pen and paper. You will, finally, realise that you have known mathematics and that it is an imperceptible part of your daily life.

Introduction

Animals, other than humans, also use and know some mathematical skills, like research studies have shown that crows know how to keep a track of upto 30 persons. Bees can measure angles and lengths. And, almost all animals learn to recognise shapes and sizes. Rabbits need to learn the shape of a flying hawk

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so that they can protect themselves. For animals, mathematics means survival.

Some basic mathematical abilities are probably inborn in all human beings. Almost anyone can tell the difference between two objects, identify the difference in the sizes of two unequal objects, and recognise the difference between a triangle and a circle. But higher level of mathematical abilities requires training. In order to tell the difference between 12 and 13 horses or between a litre of water in a bucket and one kg of edible oil, you must learn special techniques. Mathematics education aims to build inborn abilities, and gradually, take them to higher levels.

Let us now see how one can be sure that crows count. Crows are considered a nuisance in fields because they eat plants. That is why, scarecrows are placed in farms to scare them away, or sometimes, they are hit by shotguns. However, if a crow sees a person with a shotgun, it will not enter the field until the person leaves. The crow must have some recognition of the shape of the shotgun, therefore, a farmer may build a hiding place in the field called 'blind'. Even then, crows are hard to fool. If they see a person entering the blind, they will not pound on to the field until the person leaves.

One farmer thought of an easy solution. Two persons entered the blind, but only one came out. The person, who was left would shoot away the crows when they flew into the field. The crows did not come to the field until the second person had left the blind.

Then, three persons entered the blind and two came out. Even then, the crows were not fooled. Four people going in and three coming out did not fool the crows either. This was the point when everyone became curious about how high the crows could count.

At this point, everyone became curious about how crows kept a record of the number of people. So, the farmer in-charge asked more people to enter the blind. It was not until 30 people entered the blind and 29 came out that the crows were fooled into the field. It meant that the crows had finally 'lost count'.

Counting Numbers

What is counting? Adult human beings can, usually, count up to five objects without using any special technique, and therefore, numbers up to five are called 'perceptual numbers'. A person can tell how many books are there in a stack of four or five without actually counting. Even if a stack contains six or seven books, a person must count in order to tell the exact number. Counting is done by matching each book in the stack with a number name. People learn different number names and rules for combining the names to form numbers in order from one onwards. A person may count the books by saying, "One, two, three, four, five, six, seven..." The person matches each number name with one book in the stack. It tells how many books are there in the stack.

Crows probably 'count' by mental technique that humans use for five or fewer objects. Since crows cannot use language, they have developed the ability to judge larger quantities by sight.

Number name was probably not the first method that humans used to count. Long ago, humans used sets of objects to match things they wanted to count. For example, a shepherd, who wanted to make sure that all sheep were safe for the night, would match each sheep with a pebble and keep the pebbles in a bag. Each night, the shepherd would check to see if there was a sheep for each pebble and a pebble for each sheep. Hence, the sheep were counted In that way, even though no number name was used. The matching process was more important than the use of number name.

Archeologists have found hollow clay balls filled with markers along the trade routes in the Middle East. It is believed that ancient merchants used these balls to tell buyers how many items they had sent. For example, if 17 bars of copper were shipped from one place to another, then a ball containing 17 markers would be shipped, too. When the shipment reached the destination, the buyer would break open the ball, match the markers with the number of copper bars and know if the exact amount had arrived safely.

Eventually, the markers were shown as dents on the outside of the ball, so that people could check the number along the way without breaking the ball open. The clay was baked hard after the dents were made, so that no modification could be done. Dents became the first system of writing numbers. It is also known as the 'cuneiform system'. In fact, people developed ways to write numerals before they developed ways to write words. About five thousand years ago, Babylonians used numerals that looked like the signs as shown in Fig. 2.1

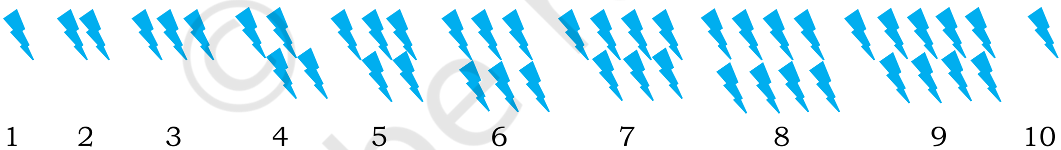


Fig. 2.1: The number symbols used by Babylonians

Measurement

The counting process results in a whole set of numbers—1, 2, 3, 4, 5, and so on and forth. These numbers are commonly called 'counting' or 'natural numbers' as it is believed that no one person or civilisation can be credited for their invention. Counting numbers are the basis of all numbers but they are not enough to solve all mathematical problems that might arise.

For example, a merchant has more than enough copper to make six bars but not enough to make seven. If he wants to ship all the copper, he would need a way to show that he is sending six whole bars and one partial bar. How would he convey this information to the buyer? The answer is to use what today are called 'fractions'.

Fractions are numbers but they are different from counting numbers. If two partial bars of equal size make one whole bar, then each partial is a half of the whole bar. If three partial bars of equal size make one whole, then the size of each partial bar is a third. In each case, a measurement takes place. The merchant is measuring the size of the partial bar in terms of the whole number 1. Fractions, thus, allow the merchant to measure the partial quantity against the whole quantity.

Things may become a bit more complicated for the merchant. Perhaps, the amount of leftover copper he wishes to send will not 'go evenly' into one bar. For example, it will take three partial copper bars to make two (not one) whole bars. The easy solution is to use fraction.

Fraction is a way of showing a relationship between two numbers—the number of parts and that of wholes. If you had any difficulty following the example of the merchant, try this: divide a chocolate bar and give your friend one half. He has half of the whole bar. You split the bar down the middle. Neither one of you is confused because fractions are a natural part of the way you think. You share via fractions.

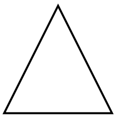
One number in a fraction tells in how many parts of equal sizes the whole bar was divided into. The other numbers tell how many parts are in the piece being measured. Here is a new way of thinking about the merchant's copper bars and using fractions to split the bars mentally.

Can you split a candy bar into halves? Thirds? Fifths? Then, you can use fractions correctly and understand the mathematical concept ratio. A ratio between two quantities is the number of times one contains the other. Since fractions show a ratio between two numbers, mathematicians call fractions 'rational numbers'. You do not need mathematicians to explain this concept, as you have been using it for years.

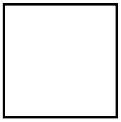
The measurement of any quantity is being done by using counting numbers or fractions in terms of some quantity called 'basic unit'. For example, a piece of length is to be taken as a unit and all lengths, then, can be measured using whole or part of this unit, likewise, all other quantities, such as mass, area and volume or capacity.

Shape

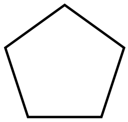
Shape is an important concept in mathematics. Shape can be defined in terms of numbers. Can you think of a shape that has three sides? Four sides?



Triangle



Square



Pentagon

All these shapes are associated with numbers. A figure having three straight lines has to be a triangle. It cannot be a square. Four straight lines of equal length make a square but never a pentagon. The five sides of a pentagon cannot be put into the shape of a triangle. Each figure has its own characteristics, which mathematicians call properties. The properties vary from figure to figure.

Take three sticks and fasten them at the ends (note that the sticks are of equal length). You have just made a triangle. You can try pushing it into different forms but it does not change. It is rigid. Now, add a fourth stick. Push the sticks into different positions. Do you always have a square? No. Sometimes, you have a parallelogram too. What does this mean in terms of numbers? It means that if you have a shape made of three straight lines, you are going to have a triangle no matter how you try to move the sides around. If you have a shape made of four straight lines of equal length, you may have more than one type of a four-sided figure. This difference in properties between the triangle and the four-sided figure is intricately interwoven with numbers 3 and 4.

Measurement enters into shape as well. If all sides of a four-sided figure made of straight lines having the same length, the shape has one set of properties. Else, the shape has another set of properties. The same goes for triangles.

A triangle measuring 3 cm, 4 cm and 5 cm is an important shape in mathematics. Any triangle whose sides have the measurements 3, 4, 5 — no matter what the measurement units are — always makes an angle of the same degree between the three-unit side and the four-unit side. This angle is called the right angle, and thus, the triangle is called the right-angled triangle.

The properties of the right-angled triangle have interested mathematicians for thousands of years, like Bodhayan and Pythagoras.

Patterns

Through ages, people did not have to go to school to see that number and measurement were closely related. They saw that there were patterns in counting and measuring physical objects. For example, a pair of shoes and twin eyes both mean two objects, but no one ever says "a twin of shoes". In a certain North American Indian language, different number words are used for living things, round things, long times and days. Fiji language uses one word for 10 coconuts and another for 10 boats. These words developed without the basic pattern involved in 'twoness', 'tenness', 'hundredness', or number in general.

Similarly, with the triangular shape, what is important is not what the triangle is made of—just as twoness does not depend on whether the objects are shoes or eyes. In fact, people began to think that a triangle, like a number, was a pattern.

Many conclusions were drawn from this observation. Both number and shape have been dealt with in mathematics because both follow patterns. In other words, mathematics is the study of patterns, and the study of patterns is mathematics.

Logic and Proofs

Mathematics has some fairly obvious patterns. Consider a pattern, such as the following:

$1 + 3 = 3 + 1$ $\quad$ $11 + 5 = 5 + 11$

$47 + 38 = 38 + 47$ $\quad$ $332 + 6 = 6 + 332$

You can observe that this pattern holds true for many pairs of counting numbers. But no matter how many pairs of numbers you check, there will be pairs that you have not checked. If you want to be sure that the pattern holds true for all pairs of counting numbers, you must go beyond simply seeing that the pattern is true for a number of pairs.

One of the convincing ways to ensure that the pattern is true for all pairs of counting numbers is to use logic. Logic is also called 'reasoning'. Logic is an argument that as one set of conditions is true, a given result must follow. For example, if you know that

All human beings are mortal.

Suresh is a human being.

Then, you also know that.

Suresh is mortal.

This example is a kind of logical scheme of formal argument called 'syllogism'. But the arguments of logic can be less formal than that. For example, a multiplication problem, such as 4×5, is shown as a set of beads, four rows each with five beads (Fig. 2.2b).

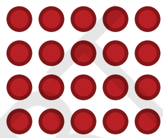


Fig. 2.2 a

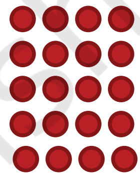


Fig. 2.2 b

You can also show the problem 5×4 with five rows each with four beads (Fig. 2.2a). If you turn the second arrangement, you will find the first one. Obviously, turning the arrangements does not change, therefore, $4 \times 5 = 5 \times 4$. The same reasoning will apply to 37 beads, 48 beads, and in fact, to any number of beads in arrangement of rows and columns. This line of thinking is a proof of the fact that for counting numbers, called 'n' and 'm' here, it is always true that

$$n \times m = m \times n$$

It does not matter which two counting numbers 'n' and 'm' are.

Logic is the main tool for finding patterns but is not the same as mathematics. Logic by itself, however, does not go far enough. More than 2000 years ago, during the period of the ancient Greeks, mathematicians had tried to set up perfect rules for logic and math — rules that everyone could agree with. Then, it would be possible to say what really was a proof and what was not. For example, how do you know that turning the arrangement does not change the number of beads? Should turning the arrangement be accepted as a proof — a legitimate way to solve problems in math?

Greeks believed that there were a few simple rules of logic and math that everyone could accept and they called the rules of logic 'axioms' and those of math 'postulates'. This idea helped in establishing the truthfulness of many assertions. For example, when applied to the study of shapes, Greek mathematician Euclid (305–285 B.C.) was able to show that about five axioms and five postulates were enough to prove everything that was known (later, mathematicians improved on his system, but not on the basic idea). This approach to mathematics is called an 'axiomatic system'. As a result of Euclid's success, it became common to think of proof as something that happened only in axiomatic systems. But in reality, early mathematicians proved results in whatever ways they could.

Counting, measurement, shape, patterns, logic and proof — all are parts of math that are easy to think about. The ideas discussed above are common and used in our daily life. They became a part of our life, language and tool to solve problems. Our thinking patterns have acquired these ideas in a way that lead to most of our conclusions. In other words, mathematics in our mind is what we use and dwell on further.

REFERENCES

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