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Understanding Numbers — Concepts and Some Misconceptions

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Abstract

Interactions with students and teachers in NDMC schools reveal that there are some problems in the understanding of numbers during teaching-learning in classrooms. This paper tries to address the importance of numbers in mathematics, as well as, in our daily life. Though numbers are an essential part of our day-to-day life and mathematics cannot exist without numbers, students and teachers still have misconceptions about numbers. This paper attempts to discuss with the help of examples some of the problems faced by primary level children and teachers.

Introduction

"Without Mathematics, there is nothing you can do. Everything around you is mathematics, everything around you is numbers."

— Shakuntala Devi

This quote of Shakuntala Devi presents a picture about the importance of mathematics and numbers in our life. Numbers are an essential part of mathematics and a large portion of the subject is developed around numbers. Can we imagine our life without numbers? If there is no number in our life, then how will we decide the date and time? How will we determine the route number of a school bus or bus to our workplace? How will we decide our age? How will we distinguish and identify a car without a number plate? What will be the form of the currency? How will money be transacted? Think of life without numbers. You will find that life is certainly impossible without numbers.

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When we talk of numbers, we, generally, think about 1, 2, 3, 4... In mathematics, there are so many numbers — whole numbers, natural numbers, integers, rational numbers, irrational numbers, real numbers, complex numbers, etc. But when we talk of numbers, the general perception that comes in the mind of a common man is 1, 2, 3, 4..., i.e., numbers, which are generally used for counting. People, while talking of numbers, seldom think about rational, irrational or complex numbers. One of the reasons behind this may be that everyone is not skilled enough to think about these. But what about people who have the knowledge of all types of numbers but still think like a layman when asked about the same.

Another important point that we would like to emphasise here is that when asked about numbers we normally think of natural numbers. Most of us remain confined to positive numbers and some of us may think of negative numbers. But what about 'zero'? How many of us think about 'zero' at the first instance when we think of a number? Take a sample of 10 people from different age groups, educational and socio-economic backgrounds, and ask them to think about a single-digit number and write it down. You will hardly find that anyone has written 'zero'. What can be concluded now? Is 'zero' not a number, or is there a misconception about the status of 'zero' — whether 'zero' is to be treated as a number or not?

To bring more clarity in the aspect which we want to discuss here, take an example — class teachers of a primary school were given the task of noting down the birthday month of each student in their respective classes and prepare a table with the name of the month in one column and the number of students whose birthdays fall in that month in another column. Suppose, there are two students in a class, whose birthdays fall in January. Then, write 2 against January, and so on. If no child's birthday falls in a month, write '0' against it. A sample of the table prepared by a teacher may be as follows:

S. No.	*Name of the month*	*No. of students having birthdays in a month*
1.	January	02
2.	February	05
3.	March	04
4.	April	04
5.	May	00
6.	June	01
7.	July	03
8.	August	02
9.	September	04
10.	October	02
11.	November	03
12.	December	05

In almost every class, there was a '0' in the list, meaning that there were some months, in which no child was born in a class. The teachers are then asked to tell the month having the minimum number of birthdays. Most of the teachers got confused. They were not sure whether a month having '0' birthday would be the answer. For example, the teacher, who prepared the table (see p.28), was confused whether the month having the least number of birthdays would be May or June. The teacher thought that having '0' birthday in a month meant having no birthday in that month. Therefore, the month having the least number of birthdays would be June, in which only one child's birthday fell. On the other hand, '0' it must be noted that '0' is a number. Therefore, the month having the least number of birthdays should be May. Does '0' actually mean 'nothing' or is it something different from 'nothing?

Take another example — ask a simple question to students. How many numbers are less than 10? Some would say 9, counting the numbers from 1 to 9. Some would say 10, including '0' as one of the numbers. Some may delve deeper and say infinity, including the negative numbers –1, –2, –3... as well. While talking about numbers, we confine ourselves to integers and do not bother about other numbers.

It is natural to ask why there is confusion or variation in our perception about '0' and about the negative numbers, or broadly speaking, why we do not have a clear perception of the entire number system. To remove ambiguity in the perception of numbers, we must understand that numbers are used in different forms. The perception or understanding of numbers may be different with change in its form. There are three forms in which a number can be used — cardinal, ordinal and nominal. Cardinal numbers show quantity and are also known as 'counting numbers'. Cardinal numbers tell 'how many', for example, five children, four computers, nine players, etc. Ordinal numbers show the order of things in a set, for example, first, second, third... Ordinal number shows only the position or rank and does not indicate quantity, for example, third child from the left in the first row, second largest country, etc. Nominal number is used to name something. Nominal number is neither used for counting, nor for indicating a rank or position. It is used to name or identify something, for example, a player wearing jersey number 99, a car having number 0623, an office in an area having postal code 110001, etc.

Consider the following sentences:

1. Route number 6 bus is my school bus.
2. There are six buses in this route for my school.
3. The sixth bus from the right is my school bus.

Here '6' has been used in all the above sentences to denote a bus. In sentence 1, '6' is used as a mark of identification for the school bus to

distinguish it from others. Thus, 6 is used here in the nominal form. In sentence 2, '6' is used as a cardinal number because it indicates the number or quantity of the buses. In sentence 3, '6' indicates the position of the bus and is used as an ordinal number. Numbers are used as labels for identifying things (nominal aspect), putting things in order (ordinal aspect) and as indications of how many are there in a set of things (cardinal aspect) [Haylock and Cockburn, 2003].

Haylock has pointed out that some aspects related to numbers cannot be understood, if we think of them in the cardinal sense as a set of things. We have to establish a connection with numbers used in the ordinal sense, as labels for putting things in order. For example, if we talk about a bus having route number 6, the number is mentioned only to identify it from other buses. It is not the case that the bus will follow immediately after the route number 5 bus. In the example given above, when the teacher was asked to point out the month having the least number of birthdays, s/he was perplexed in giving a reply because s/he might have interpreted the situation in two ways — '0' birthday in a month practically means no birthday in that month. Therefore, May cannot be said as the month having the least number of birthdays as no birthday fell in that month. But from the aspect that '0' itself is a number, in fact it is the least number of the set of numbers formed in the example, the answer should be May.

Haylock has explained the difference between cardinal and ordinal numbers. For further reading on the subject, we recommend Haylock and Cockburn. If one is able to understand the difference between cardinal, ordinal and nominal forms of numbers, there will be no problem in describing the numbers and their use.

Infact, '0' is a 'well-decorated' number of the 'number system'. We all know that symmetry has an important place in the study of science and mathematics. Be it the field of architecture or drawing and painting or interior decoration, symmetry gives elegance and perfection. For example, take body parts that are in pair, such as ears, eyes, eyebrows and to some extent nose (pair of nostrils). Any asymmetry in these pairs may lead to ugly look, whereas, symmetry makes our look perfect. For people who love symmetry, '0' is a 'well-decorated' number in the sense that it is the point of symmetry of the Real Line. It occupies the central place Real Line and the position of numbers is the same on both of the sides of '0'. The Number Line starts from '0'. At one side of it positive numbers are placed, whereas, the negative numbers are placed on the other side.

In our daily life also '0' is given a special treatment — '0' floor means the ground floor. There is a section in our society, which likes to live on the ground floor. Ground floor of a multi-storeyed mall or store is normally

different from the other floors. It is well-maintained, attractive than the other floors, displays pictures of items available in the store and their price range, provides a detail about the store, offers for customers, etc.

Despite all this, '0' has been getting secondary treatment. Most of us are not ready to treat '0' as a number. Our perception about '0' is not mathematical. Not only '0' but the negative numbers, too, are not understood well by most of us.

Haylock emphasises that the ordinal aspect of numbers may be introduced to students in the primary classes itself, so that the possibility of getting confused in later years is reduced. If I have five chocolates and I distribute these among five children, how many chocolates am I left with? This is the most common way to introduce the concept of '0' to children in primary classes. However, if '0' is introduced like this, how will one introduce negative numbers? What is the meaning of –5, how will you explain it? One of the practical approaches to introduce negative numbers is to take your child to a lift or an elevator. The ground floor is denoted by '0', the basement (upper ground floor) and lower basement (lower ground floor) are denoted by –1 and –2, respectively, and the first, second, third... floors are denoted as 1, 2, 3... in the control panel of the lift. Here, the child can learn the concept of '0' and negative numbers in a practical way. The use of Number Line and the concept of negative temperature may be useful in helping them understand the concept of negative numbers.

Number sense describes the intuitive idea about number understanding and its use in different forms, and one's ability to differentiate between these forms. According to Gersten and Chard, number 'sense' means students should have a sense of what numbers mean. They must understand their relationship with each other and be able to perform mental math by understanding symbolic representations and use those numbers in real life situations. Advocating the relevance of 'number sense', Carlyle and Mercado state in their book that it is important as it encourages students to think flerxibly and promotes confidence with numbers — they make friends with numbers.

The National Council of Teachers in 1989 identified the following five components that characterise number sense:

- number meaning
- number relationships
- number magnitude
- operations involving numbers and referents for number
- referents for numbers and quantities

Hence, numbers are an essential part of our life and we cannot imagine life without numbers. Similarly, mathematics also cannot survive without numbers.

REFERENCES

Burns, Marilyn. 1997. How I boost my students' number sense. *Instructor Magazine* (April). pp. 49–54.

Carlyle, Ann and Brenda Mercado. 2012. *Teaching Pre-school and Kindergarten Math: A Multimedia Professional Learning Resource*, Math Solutions, Sausalitolito, CA. 94965.

Haylock, Derek. 2010. *Mathematics explained for Primary Teachers* (4th Ed). SAGE Publications, London.

Haylock, Derek and Anne D. Cockburn. 2003. *Understanding Mathematics in the Lower Primary Years: A Guide for Teachers of Children.* SAGE Publications, London. pp. 3–8.