Haneet Gandhi
haneetgandhi@gmail.com

What Mathematics Matters to Teachers?

Abstract

This article draws attention to the need of mathematising the teachersof mathematics. The paper questions and seek answers to the didactical approaches that should be adopted to engage teachers in acts of thinking mathematically.One of the proposed ways is by challenging teachers' existing mathematical cognition in a constructive manner. The paper further elaborates a task that was instrumental in setting up conditions for thinking, reasoning and making conceptual connections.

When the NCERT's National Focus Group took a position on teaching of Mathematics (Position Paper on Teaching Mathematics, 2005), they explicated the higher and narrower aims of learning Mathematics, emphasising teaching to be central around 'mathematising' the child's mind. Indeed, undoubtedly the document, though written perspicaciously, falls short in defining the knowledge, skills and preparation required on the part of teachers for meeting such goals. We question if our teachers, both in the service and preparing for the service get enough opportunities to mathematise themselves.

Through this article, I wish to focus on the preparation of Mathematics teachers (through Teacher Preparation Programmes) and of their continual learning (through Professional Development Programmes) as a means of getting mathematised. I question if our teachers are mathematised enough. Do they understand acts that lead to 'mathematisation'? Only when the teachers are exposed to the strategies embedded within the frameworks of promoting mathematical thinking, they will be able to adopt similar routes in their classrooms. My proposal is to question and seek answers to the didactical approaches that we, as Mathematics Teacher Educators need to implement to let our student teachers, who are already familiar with the content matter, approach the subject from a wider perspective, one that is needed for teaching children effectively.

There are many opportunities for educating teachers. At the entry stage, the Institutions such as CTEs, IASEs , DIETs, SCERTs and other teacher training institutes provide the first course on teacher preparation. The in-service, teachers get several opportunities to revive their skills and knowledge through Professional Development Programmes (PDPs). The arguments and proposal that I am going to present in this article hold for all such programmes. I am going to argue that revisiting mathematical content, judiciously, at all levels of professional engagements is not only important, but also challenging for Mathematics Teacher Educators.

I will refrain myself to comment on the content knowledge that the entrants of a teacher preparation program come with. This requires a detailed analysis. However, through my informal observations, while engaging with them, I understand that at times the prior understanding of students who come to the teaching profession is so shaky that in no case it can be rectified in a one or a two-year teacher preparation course. The pre-service teachers, though graduates and post-graduates in Mathematics, face difficulties in learning and understanding the fundamentals of the school content. The Mathematics department does not teach them this Mathematics and definitely, there is no scope to redo all the required Mathematics in a curt two-year preparation course.

The challenges of in-service teachers are somewhat dissimilar. Teachers being burdened with multiple responsibilities, trying to complete courses within rigidly limited time, being one of them, often leaves them bewildered. Since school assessment is predominantly based on quick recall of correct procedural know-how, the teachers do not feel a need to move beyond certain shortcuts and routine algorithms. I think the teachers also get conditioned over the years. They recognize what is significant (in terms of assessment only) and this limited view sets their teaching objectives. I will not elaborate how such a view harms children's learning as many have expressed this at various platforms. Further, I understand (again through informal talks during PDPs) that as teachers gain experience, they dissuade gaining new insights. What is needed in such programmes is to produce opportunities that evoke a sense of challenge in the teachers. The tasks should urge them to learn the basics, which get missed in the course of teaching concepts repeatedly.

In a nutshell, pre-service teachers have never dealt with the content that is needed for teaching and those inservice feel that looking back at the content is a fruitless exercise. Either way, the call is on rebuilding the mathematical knowledge of teachers.

Different people interpret the nature of the content needed for Mathematics teachers differently. Some consider it as revisiting the school content while for others it means going beyond school Mathematics. What sort of Mathematics do teachers really need to know is undefined or at least unfamiliar to many. What Mathematics matters to them? A knowledge of content is as significant as the pedagogical ways of transacting it. The study of some researchers such as Shulman, followed by Ball, Hill, Adler and their colleagues (to mention just a few) provides a starting point to building teachers' content and pedagogical understanding. They state cogently that preparing for a profession, especially of teaching, requires the amalgamation of content and a judgement of appropriate teaching practices. These researchers extensively elaborated on the tasks which teachers need to engage with and those that can be engrafted in their classrooms.

I go further to say, the pre-service teachers must also be provided with opportunities for thinking mathematically. The opinion is on necessitating teachers to reason mathematically. To bring faith in their practices, the teachers themselves need to be engaged in acts of conjecturing, communicating, reasoning, debating and making connections. It's akin to making teachers as agents of creating Mathematics. In other words, teachers need to make sense of the mathematical activities themselves.

One of the ways through which I try to encourage my students to (and teachers in PDP workshops) revisit the content is by 'challenging their cognition'. The didactic intention is to present the content that questions their mathematical knowledge in a constructive manner. I understand that it is not easy to change teachers' beliefs and methods. It calls for shaking their existing beliefs, schemas, practices and methodologies. Of pertinence here is prefixing the verbs of learning with 're', re-look, re-learn, re-conceptualise and re-engage with the concepts. The teachers should be afforded with the same experiences as their students are likely to get when they come across a new concept, process or idea. Pre-service and in-service teachers should get opportunities to re-learn and re-conceptualise the Mathematics. The mere presentation of school mathematical content or just good examples of teaching does not avail. Create situations that make unfamiliar. Teachers' training in the content should be an exercise of creating conflicts in their existing cognition. The didactic approach should try to capture, to some extent, the issues that make learning meaningful and which arouse alertness to students' difficulties.

Several arguments support the proposed didactical approach. To begin with, teachers need to be aware of the learning gaps and difficulties that their students face when they come across a new idea. That is, teachers need to view Mathematics from the same platform as their students do while learning a new concept. This is possible when new experiences are provided to the old content. Secondly, teachers' own experiences of learning Mathematics are eclipsed by procedural approaches. By challenging teachers' existing practices a strong need to rivet on the underlying conceptual knowledge would emerge. Lastly, it is hoped that by delving into such activities, teachers would become better observers of their students' work.

Designing appropriate tasks through which the content and pedagogy understanding can be amalgamated is not easy. While designing such tasks my intention is to engage teachers (and students) in reflective processes. In this article, I'll present one such exercise where my students (and teachers) got opportunities to re-look at a concept as a content building exercise. The example draws learnings from integrating the history of the evolution of mathematical concepts. The tasks set up the conditions for thinking, reasoning and making conceptual connections. The illustrated example is one of the many instances of a more general didactic approach.

Using the History of the Evolution of Concepts

In this section I will describe a lesson that integrates the journey of number formation, starting from the fundamental acts of enumeration to systematic approaches of extending numbers and creating Numeration systems.

To start with, I frame a set of inquiries that provide me with a lead: What is it about Numeration systems that the teachers need to know? What are the central characteristics of the current Numeration system? How did humanity arrive at these fundamentals? What aspects does a teacher need to be alert with while building the idea of the place - value system? Which pre-concepts are needed? These questions helped me in picking out moments of importance in the evolution of Numeration systems. Though the above thoughts assisted in planning for teachers, they are, to a large extent, guided by the learning difficulties that children face while working in the place-value system.

Indeed, I have to think of the areas that would kindle teachers' attention. Presenting history could at times be dull and boring. The concern was to present story as a concept building exercise. Conscious efforts were made not to present the material in a chronological way or as biographies (of mathematicians, as presented in textbooks). It was ensured that the concepts bind themselves to form a meaningful sequence, rather being presented as disjointed chunks of information. Thus, instead of approaching History as a chronology of events, the tasks were arranged to bring out a conceptual-chronology. Moving from the easiest to the complex numeration systems. The intention was not a mere transmission of the historical facts. The material should let teachers construct their ideas and make sense of the fundamental processes that form a numeration system. It was kept in mind that during this course the teachers should be able to deduce linkages between the various numeration systems. Concomitantly, the readers also had to bring out the shortcomings of a numeration system and the determinants that led to the systematisation of others. This requires a reflective mode, which is also flexible. The teaching materials were designed to evoke reasoning.

The Material

To understand the basis of the current Numeration system, i.e. the Hindu-Arabic system, two fundamental concepts need to be established: the positional feature that assigns value to a number and the base number that forms grouping. The journey of arriving at these two concepts is elucidated through the modules.

The first set of module presents the genesis of counting as a need for keeping records. The first set of worksheets introduces the reader to the primitive ways of counting. The objective is to evoke students' attention to how humans made connections with the principle of one-to-one correspondence and used the ready-to-refer material such as body parts to do so. The module also brings to the fore the idea that in the early stages the concept of numbers did not imply nor was there any necessity to have them. The thought of expanding numbers evoked the need for grouping. This idea is brought up in the next set of worksheets. In this set, the transition to Numeration and number words is covered. As part of a reflective exercise, the students have to deduce mechanisms of expanding numbers beyond finite counters. This exercise evokes a need to systematise the expansion of numbers to extend them to infinity. This exercise culminates in a reflective discussion elucidating the advantages of creating a Numeration system over eNumeration.

For a more formal growth of numbers, numeration systems emerged. The second set of the module covers the numeration systems of five influential civilisations: Egyptian, Babylonian, Roman, Greek and Mayan, in this order. The module presents an elaborate description of the Numeration system of these civilisations.

The structure of the worksheets on numeration systems is similar. Each module comprises an introductory phase and a chart demonstrating Numeration system of a civilisation. The students have to study the system and conjecture the rationale behind the processes that would have contributed to the idea of systematising the process of Numeration in the given civilisation. The students are encouraged to find the grouping number, basis of grouping and the symbolic representations of the numbers of the respective civilisation. To give them hands-on experience, they are encouraged to construct numbers abiding to the rules of the system. This exercise acquaints the readers to the rules and syntaxes.

As part of the next exercise, the students have to work on basic calculations of addition, subtraction, multiplication and division on the numbers they had formed during the previous exercise. This activity often turns out to be quite hard one. Working on a new system is challenging. It's not easy to forget the old rules and learn new ones. We have been conditioned to working in the Hindu-Arabic system, so our habituated mind dissuades learning the new format. Performing calculations on the new system challenges the existing cognition. While playing on the arithmetic operations the teachers (and students) face, for the first time, situations that shake up their existing knowledge. It draws their attention to the processes that people of particular civilisations could have adopted. Later, after much practice, as a follow-up, they are asked to develop the algorithms for calculations. At this point one sees teachers reasoning, making arguments, conjecturing the possibilities and convincing their colleagues on the key features of the concept in hand. All acts that lead to thinking mathematically are evident in these sessions. The activity deconstructs several ideas and this deconstruction, in turn, establishes a profound understanding of some of the most elementary concepts such as of grouping, selecting a suitable number for grouping and determining the value of a number based on its position. Finally, after achieving enough acumen with a particular numeration system, as a reflective exercise, the students (and teachers) are required to compare the various numeration systems and bring out the similarities and differences. These reflections leave a deep impact on the teachers' (as well as students') learning.

Wherever available the material is also supplemented with videos. The worksheets are made in self-explanatory mode. The participants work in pairs and the results are shared with the entire group. We pause occasionally to share our understanding, seek clarifications and put forward the deductions. The solutions are never stated directly, but are worked out collectively as a group. The whole group discussions give scope for larger debates and consensus. Since none of us know the grounds behind the development of a system, we all make several conjectures, some mathematical, some not-so-mathematical, but interesting.

By-products

At the beginning the material on the history of the evolution of numeration systems was prepared with a prime agenda of challenging teachers' beliefs and methodologies on the current place-value system. We are glad to share that each time this lesson is executed many other ideas and concepts emerge as by-products. Among these, the most frequent one that comes up virtually in all the discussions is about the genesis of zero. Discussions regarding the existence and nonexistence of zero in a numeration system are most frequent. Arguments considering zero as a void or as a placeholder or as a cardinal number emerge invariably and often lead to long discussions and debates. Teachers are seen debating, sharing thoughts, arguing and attempting to provide convincing reasoning to their ideas regarding the role and value of zero. These discussions always come as residual and a much-valued one. I call it a bonus!

Knowing history gives a pretext and a context. When one studies the growth of an idea, one gets to appreciate its emergence as a product of cross-generational and cross-conceptual confluence. This is the second by-product. While walking through the galleries of history, teachers and students try to decode the cultures and social norms of the civilisations. Along with the advancements in Mathematics, the readers also visualise the subject as a cultural product, one that is created by people, at a particular time, attributed to the then existing needs. They appreciate the ever-evolving character of Mathematics as an enterprise of human minds.

Finally, being open-ended, the activities encourage a research-based learning, giving a mathematical ownership, helping students and teachers profit mathematically.

Some Closing Thoughts

As a concluding comment, I urge the Mathematics Teacher Educators to create situations that mathematise the Mathematics (as Freudenthal calls it) for the teachers. Such approaches, I trust, will serve in preparing teachers to become more serious observers, who are receptive to listening and respecting their children's ideas. By becoming agents in the process of creating Mathematics, the teachers will understand the subtleties of 'thinking Mathematics'. By reflecting on their difficulties, teachers become conscious of the difficulties that their students are likely to face.

I am aware that the assertion I am making is not new to most of the Mathematics teacher educators, yet re-educating teachers in a sense-making activity, appropriate to their level, is a challenge for many, if not for most. Meaningfully mathematising the Mathematics for teachers should be a critical dimension of any teacher education programme. Summing up, we need to create situations for teachers to 'experience Mathematics' which, in turn, will strengthen their mathematical knowledge. It is these choices of experiences that make the subject memorable, one that is cherished lifelong.

Notes:

2. 1. In the B.Ed. programme of the University of Delhi only those who are graduates or postgraduates in Mathematics can opt for the courses related to Teaching of Mathematics. During their internships, the graduates get to teach middle grades and the postgraduates teach higher classes.
2. Lee Shulman pioneered the special kind of knowledge that is required for teaching. He had recognised the dichotomies that existed between content knowledge and pedagogical practices in teacher preparation programmes. He thus promoted an amalgamation of the two. He is credited with popularising the phrase Pedagogic Content Knowledge (PCK) for teachers. Following his ideas, in the field of Mathematics Education, many Mathematics educators such as Deborah Ball, Heather C. Hill and Jill Adler have done extensive work in the understanding, elaborating and categorising PCK as a construct. As a result, the idea of PCK now encompasses many dimensions; some theoretical, some radical, some controversial.
3. Here, 'students' means pre-service teachers (or student-teachers) who are enrolled in the Teaching of Mathematics course of B.Ed. programme, University of Delhi.
4. For a collection of videos on History of Mathematics, visit the BBC website. They hold a collection of documentaries on various civilisations and biographies. My personal favourite on numeration system is their documentary-Story of One. Retrieved January, 2016 from https://www.youtube.com/watch?v=qevpRffg6wc.
5. For more along this idea, refer to Hans Hans Freudenthal's article, Why to Teach Mathematics so as to be useful. *Educational Studies in Mathematics*. May 1968, Volume 1 (1), pp 3-8.

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