

## Investigating Conceptual Changes and Difficulties in Learning Mathematics during the Transitional Phase from Arithmetic to Algebra

### Abstract

*The sequential nature of mathematics is well known, and prior knowledge makes it more prominent in the process of mathematical learning, especially in the phase when mathematics shifts from the basic concepts of arithmetic to algebra. While learning mathematics students start with the basic concepts of arithmetic and then proceed to further mathematical learning. Thus, these all are dependent on each other in a sequential pattern. This study tries to identify the main difficulties faced by students in this particular phase along with the intent to explore the transitional phase of mathematics. The present study was conducted with the aim to identify the difficulties faced by students in the learning of algebra. The study has also tried to identify its causes with the help of a process-based approach.*

*Keywords: Difficulties, Algebra, Transition*

### Introduction

In the present era teaching and learning of mathematics is an issue of debate across the whole teaching and learning of algebra. Teaching of algebra offers a serious challenge due to its problem solving and symbolic issues become a major concern in the transitional phase of learning mathematics (Subramaniam & Banerjee 2011).

This study was focused on the transitional phase of learning mathematics. Therefore, Grade VI was taken for the study. In our country, algebra is formally introduced for the first time in this grade and before this class no such introduction and exposure of algebra was given in the curriculum of mathematics. Since most of the students have come with their prior knowledge of arithmetic they have studied at the previous classes. Teaching Learning of mathematics is an issue which should always be taken for the research. In the present times researches have revealed that difficulty in learning algebra proves to be a barrier in the later stage of learning higher mathematics

(Bazzini and Tsamir, 2004; Subramaniam & Banerjee, 2004.) Since algebra is the gateway for higher mathematics, difficulties in algebra ultimately add to the difficulties (Sugiarti & Retnawati, 2019). Since algebra is introduced in Grade VI in our country, prior to this students have previous knowledge of arithmetic alone. Therefore, during this phase of transition from arithmetic to algebra, it becomes necessary to understand the conceptual changes that have occurred and which kind of difficulties have been faced by the students while learning. The present study is based on these fundamental issues, that is why are students not able to cope with the problems of algebra.

### Concept and Concerns of the Transitional Phase of Mathematics

In the schooling system, the curriculum is designed with the intent to link the concepts to the maximum extent. The problems of learning in algebra arise not only due to its nature as well as traditional management of course and curriculum (Lodholz, 1990).

Various researches have shown that there is a gap between the arithmetic and algebraic knowledge and this gap creates serious problems in algebra learning (Pillay, Wilss & Boulton-Lewis, 1998). They have also suggested arithmetic and algebraic knowledge from a cognitive perspective in an effort to determine what constitutes a pre-algebraic level of understanding and also suggest a model for the transition from arithmetic to pre-algebra to algebra to prompt the students' understanding of algebraic knowledge.

According to Warren (2003), students' understanding of the associative law, commutative law, and addition and division as general processes when they have completed their primary school education. The understanding of all these related concepts have assisted in the successful learning of algebra during the transition phase from arithmetic to algebra. This discussion from makes a clear sense that after completion of primary education students understanding of basic arithmetical concepts has laid the strong foundation in the process of developmental stage of algebra have also supported the issues that during the transitional phase of the arithmetic to algebra, the identification of operators and variables with the dualities in equality (operator– equivalent) are also challenging task for the students along with the duality of zero and non-acceptance of the negative solution (Gallardo & Hernández 2005).

Research has reported that while attempting the questions of arithmetic and algebra students use a different approach. While in arithmetic students' approach can work from the known conditions and find intermediate numerical solutions to arrive at the solution to the problem, in algebra, they are faced with the challenge of understanding symbols and handling variables to arrive at a meaningful conclusion.

In India, the teaching of algebra generally follows arithmetic in the curriculum generally in class VI. Since in the learning of algebra, students' prior experience in primary school

arithmetic comes into play, therefore, it becomes necessary for the mathematics teacher and curriculum writer to understand the gap between the arithmetic and algebra. The present study is directly related to this issue that how students starts to learn and solve the various algebraic problems and which type of conceptual changes and difficulties they face during this transition phase of leaning mathematics and on the above ground the present study was taken.

### Research Questions

1. What kind of conceptual changes have occurred during the transitional phase to algebra among Class VI students?
2. What are the difficulties faced by the students of Class VI while learning algebra?

### Objective of the Study

1. To identify the conceptual changes among students of Class VI while learning algebra
2. To study the difficulties faced by the students of Class VI while learning algebra

### Research Design

The present study uses the survey method to research the abovestated questions. This is a qualitative study, where the researcher has collected data to ascertain the students' difficulties in learning, with the help of a test. Furthermore, the researcher felt that face-to-face interaction would be the most beneficial method for data collection. Therefore, a focus group discussion was conducted with students. Each group comprised 5 students was done in terms of extracting the exact figure.

### Sample

In the present study, a total of 170 students of Class VI from different CBSE board schools have been selected as a sample for the study.

## Tools Used

In the present study, the researcher has developed two tools to collect data. Both the tools were self-constructed by the researcher. Since the study was primarily based on the difficulties in learning algebra, so the researcher has constructed a test in which different items like fill in the blanks, match the column, error identification, MCQ and process orientation were asked. This test contains 14 different types of questions on algebra of Class VI NCERT syllabus.

Overall, the aim of this is to examine the process-based solution of different types of questions and learning difficulties that had occurred in the transitional phase of learning algebra while shifting from the arithmetic.

The second tool constructed by the researcher was focus group discussion (FGD). This tool was prepared for obtaining the view points of the students and experiences regarding learning algebra and difficulties in learning algebra.

This tool contains 10 different open-ended questions related to the above. Validity of both the tools was established by experts in the field of education and mathematical education.

## Description of the Tools

### Diagnostic Test

Fourteen items comprising the Diagnostic Test (DT) such as open-ended questions, fill in the blanks type questions, match the following type questions, word problems, objective type question and question based on complete the growing patterns (reasoning), all items were process orientation questions (Carpenter & Levi 2000). Overall aim of this questionnaire was examine the process based solution of the different types of questions along with the conceptual changes that had been occurred in the transitional phase of learning algebra while shifting from the arithmetic (Lian & Yew 2012 )

Item wise discussion of the test is presented in the Table 1.

**Table 1**

Question Number in DT	Content it Covers	Area which it Targets to Inspect	Key Mathematical Ability Required to Solve the Question
1.	Basic concept of constant and variables	Identification of variable and constant	Understanding the concept of constant and variables in addition to their differences in algebra
2.	Basic Structural understanding of figures	Structural arrangement	Ability to understand and execute the structural arrangement properly
3.	Equation	Formation of an equation with one variable	Ability to understand the word puzzle and formation of an equation
4.	Solving an equation for any variable	Solving an equation with one variable	Understanding the transposition rules, concept of sign change during the transposition and calculating the value of an unknown quantity

5.	Solving an equation for any variable with sign complexity	Solving an equation with one variable with more sign complexity	Understanding the difficult transpositions, concept of sign change during the transposition and calculating the value of an unknown quantity
6.	Solving an equation with same variables on the both side of the equation	Understanding of relationship between the operations and function	Understanding the concept of similar variable
7.	Solving an equation	Understanding of relation between the operations and function in a detailed process based structure	Ability to solve an more complex equation by using the understanding of the transposition and complex variable addition
8.	Variable addition and multiplication	Adding the same variables together and their multiplication	Understating of the variable addition and function of coefficient in its addition and multiplication
9.	Concept of coefficients	coefficients and its use in addition and solving the equation	Understating of the variable addition and function of coefficient in its addition and multiplication in more complex equations
10.	Word problem	Forming an equation of word problem and obtaining its solution	Thinking about word structure and applying mathematical generalisations
11.	Variable, constant, and expression	Difference between variable, constant, and expression	Understanding the concept of constant, variables and formation of expression in algebra
12.	Solving an equation	Solving an equation with particular value of an variable	Understanding the way to solve an equation for the particular value of variable, as well as putting the value of variable in an equation to solve
13.	Multiplication of different variables together	Understanding of the multiplication of variables ( $a \times b$ )	Ability to open brackets, and do the multiplication of different variables like $a \times b$ as well as making meaning of $a.b$ and $a \times b$
14.	Growing patterns	Understanding of Growing Patterns	Ability to understand, visualise and analyse the structure and patterns (pictorial-based reasoning)

### Focus Group Discussion (FGD) Schedule

The second tool was also constructed by the researcher. This schedule was prepared for obtaining the viewpoints of students in order to know the way in which they want to learn algebra, their ideas regarding algebra, how they approach various kinds of problems

and their rudimentary understanding about the basic concepts of algebra. This schedule contains 10 open-ended questions related to the above concerns.

### Data Analysis

Since data was obtained with the help of a Diagnostic Test and an FGD schedule, it was

qualitative in nature. Also, the data obtained with the help of a Diagnostic Test was based on the process-oriented responses. Therefore, a qualitative analysis of data obtained was done.

## Findings of the Study

### Findings related to objective 1

Analysis of the conceptual changes and difficulties was done with the help of a Diagnostic Test and supported by the responses of the students on FGD.

The conceptual changes that were identified from all the questions were classified in five broad categories.

**Table 2**

No.	Conceptual Changes
1.	Basic understanding of variables and their use in solving various equations
2.	Understanding of algebraic structure and knowledge of equation formation
3.	Understanding of the sequences and patterns
4.	Transposition and its use to solve equations (when the variables are involved)
5.	Use and understanding of growing patterns and generalisation of patterns

### Basic understanding of variables and their use in solving various equations

Since students have previous knowledge of addition from arithmetic and other mathematical operations, it carries forward from there to algebra. This is quite natural, but algebraic operations are not as similar as arithmetic. Variable addition and its multiplication both have the different concepts. Students did not understand the concept of variable addition and multiplication and were unable to solve the questions. Overall, conceptual changes found under this heading are changes that occur in terms of understanding the variables,

students were not able to understand that how number acts with variables and the role of coefficient in variable addition and multiplication.

### Understanding of algebraic structure and knowledge of equation formation

An analysis of the responses on the Diagnostic Test and FGD revealed that students misinterpreted every different letter to stand for a different number. They believed that different letters cannot take the same numerical value. Even students who are good at mathematics often struggle to solve word problems. To be able to form equations with the help of word, students are required to use the knowledge of algebraic structure and syntax. Conceptual change identified in this category: when words and letters are matched from left to right when transposing literal sentences to form algebraic expressions or equations students again after transposition mistake is handling the variables with constant.

### Transposition and use of its rules of to solve various equations

Brackets are an essential component of mathematical notation in both arithmetic and algebra. However, algebra requires more flexible understanding of brackets. In arithmetic, brackets are generally used as a sign to indicate which operation is to be performed first. The order of operation plays a key role in simplifying expressions and equations.

Many students find it difficult to understand brackets in algebra. Therefore, students simply overlook it. To some students, parenthesis means just a cover of variables in algebra. This is another conceptual changes identified related to parenthesis, transposition and its use.

### Use and understanding of growing patterns as well as in the context of the generalisation of patterns



Under this category, the difficulty that students face is related to understanding a pattern. As we know, the general arrangement of numbers is different as compared to structural arrangement. Here maximum students have used a good reasoning ability but, here students have to produce a general rule to express a relationship between different patterns. Along with this, generating a symbolic expression for the general term of patterns is also required. Therefore, as observed here too, there were conceptual changes as well as generalisation of patterns, which was misunderstood by students.

### Findings Related to Objective 2

On the basis of the responses obtained by the students and its analysis, the difficulties faced by students have been classified into five major categories. These difficulties have been reported on the basis of the response and solution of the question given by the student in the test. These are broadly classified in five different categories.

1. Difficulties related to use of operator and symbols
2. Difficulties related to formulation of algebraic expression or equation
3. Difficulties related to understanding of variable and constant
4. Difficulties related to structural understanding of figures/growing patterns
5. Difficulties related to solving an equation

S. No.	Concept/ Topic	Difficulties Faced by Students
1.	Use of operator and symbols	Most of the students faced difficulty in understanding the concept of equal sign in an equation.
2.	Formulation of algebraic expression or equation from information presented in words	Question 3 and 10 primarily focused on this concept. Most of the students were unable to understand the information and form the equation on the basis of it.

3.	Difference between variable, constant, and expression	Most of the students suffered to identify the expression, but maximum students were able to identify the constant and variable.
4.	Basic structural understanding of figures / growing patterns	Here too, most students identified the structures and figures, and applied their reasoning to arrive at the conclusion, but in some cases, students misunderstood the question and were unable to recognise the patterns and solve it
5.	Solving an equation with same variables on the both side of the equation	Students struggled to understand the process to arrive at solutions.

### Analysis of Responses of Students obtained through the FGD

Overall, the responses of students obtained through the FGD revealed that students have basic misconceptions regarding algebra. Apart from this, students have also reported that while solving most of the equations of algebra, they were confused when they had to deal with parenthetical elements like  $(x + 7y)(x + 2)$ . Another aspect of learning algebra which they reported was understanding the concept of variables and constant, especially the difference of operations on constant and variables.

During the discussion with students, the most important issue identified was that they felt a difference between algebra and arithmetic, and it was found that there was a kind of cognitive gap between the two branches of mathematics (significant issue of transitional phase of learning mathematics).

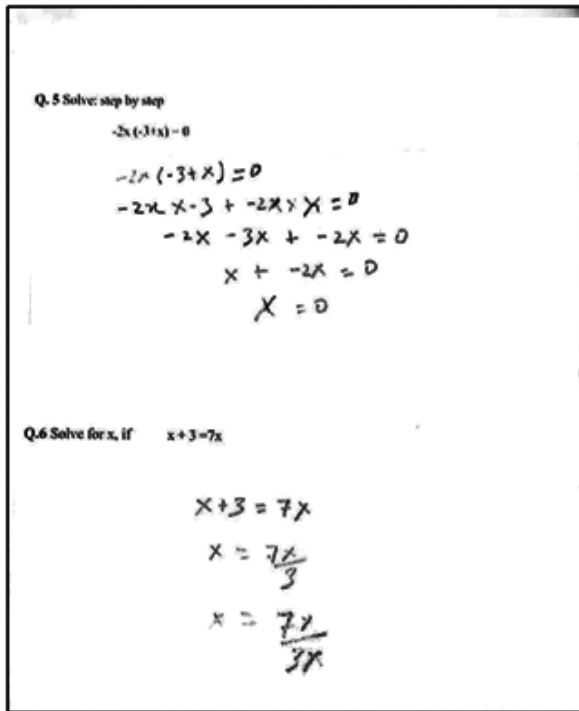


Fig. 1

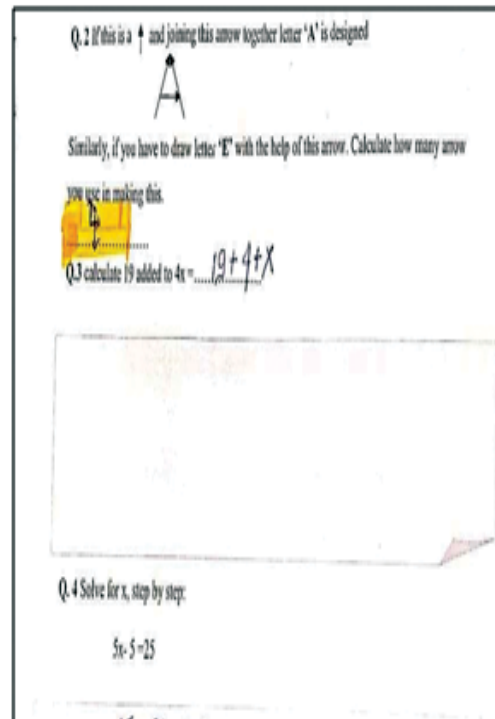


Fig. 3

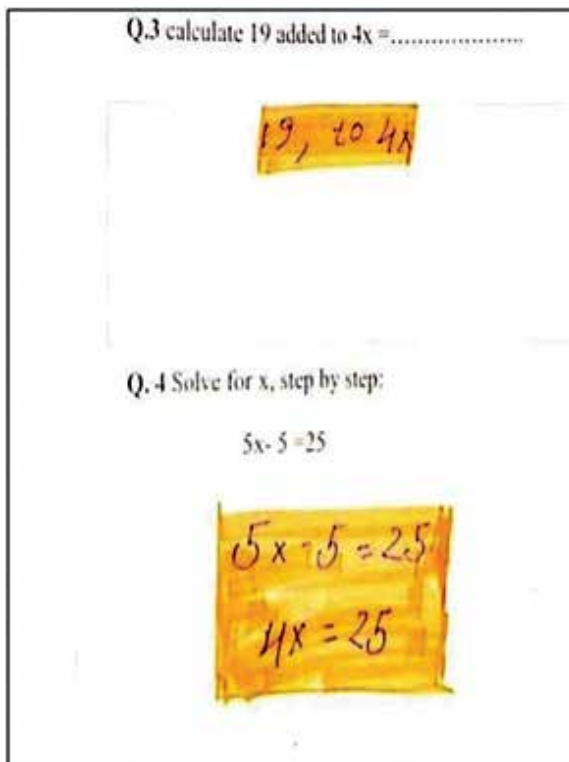


Fig. 2

Some key responses of the students

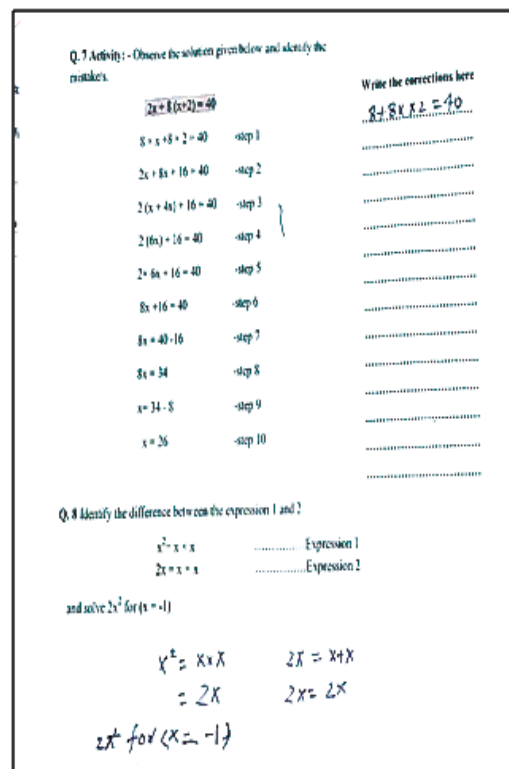


Fig. 4

## Discussion

The findings of the study reveal that students have some major difficulties in learning algebra. Problems in learning algebra are not only due to the nature of algebra, but it also seems to be influenced by the conceptual changes occurring during this transitional phase. During the transitional phase from arithmetic to algebra, students have a basic understanding of arithmetic to handle algebra. These newly building concepts require algebraic thinking to cope up with this new learning situations (Blanton, M. et al. 2015, Subramaniam & Banerjee 2011). The recorded responses have also attracted attention towards the different types of difficulties faced by the students. These difficulties have occurred probably due to the symbolic nature of algebra and its abstractness. Few mistakes made by students were due to their arithmetic knowledge, which they have applied to handle the variables, constants and equations, understanding of word-problems and articulating them into the equation, which seems to be a major challenge for students.

## Conclusion

Most of the students were facing the difficulty in understanding the concept of equal sign in an equation. Students have also suffered to identify the expression and use of transposition in an equation, but most of the students were able to identify constant and variable. Some difficulties were identified in

recognising the patterns and solving it, but most of the students were unable to solve the equation. Overall, process based questions have exposed difficulties related to the functional issue of operators while solving the various question of algebra (Koedinger & Terao, 2002). These issues can be kept in mind while developing a lesson plan and regular classes of algebra, especially at class VI level. At this level algebra appears to be a generalised form of arithmetic. Due to this, prior knowledge of arithmetic always plays a vital role in algebra learning. Since, the numbers are still used but their role are changed when dealing with the variables. Therefore, with the use of understating of symbols and way of its use along with constants and variables in the algebra becomes very prominent to be understood by the students.

## Educational Implications of the Study

The findings of the present study can be utilised as:

- Diagnostic teaching can be developed on the basis of identified conceptual changes and difficulties.
- Teachers can use the result in their regular classroom teaching.
- The identified student's difficulties and conceptual changes in the transition phase of learning mathematics can be utilised to overcome the challenges of algebra learning.



## References

- BAZZINI, L. AND P. TSAMIR. 2004. Algebraic Equations and Inequalities: Issues for Research and Teaching. In Proceedings of the 28th Conference of the International Group for the Psychology of Mathematics Education (pp. 152–155). Bergen, Norway.
- BEATTY, R. AND C. BRUCE. 2012. From Patterns to Algebra: Lessons for Exploring Linear Relationships. Nelson Education, Toronto, ON.
- BEST, JOHN W. AND JAMES V. KAHN. 2007. Research in Education (9th Ed.). Prentice Hall of India Pvt. Ltd., New Delhi.
- BOOTH, L. 1988. Children's Difficulties in Beginning Algebra. In A. Coxford (Ed.), The Ideas of Algebra, K–12 (pp. 20–32). National Council of Teachers of Mathematics, Reston.
- CAI, J. 2004. Developing Algebraic Thinking in the Earlier Grades: A Case Study of the Chinese Elementary School Curriculum. *Mathematics Educator*. Vol. 8, No. 1. pp. 107–130.
- CAI, J. AND E.J. KNUTH. 2005. Introduction: The development of students' algebraic thinking in earlier grades from curricular, instructional and learning perspectives. *ZDM-The International Journal on Mathematics Education*. Vol. 37, No. 1. pp. 1–4.
- Carpenter, T.P. and L. Levi. 2000. Developing Conceptions of Algebraic Reasoning in the Primary Grades. Res. Rep. 00-2) . National Center for Improving Student Learning and Achievement in Mathematics and Science, Madison, WI.
- Gallardo, A. and A. Hernández. 2005. The Duality of Zero in the Transition from Arithmetic to Algebra. *International Group for the Psychology of Mathematics Education*. 3 , pp. 17–24.
- KIERAN, C. 1992. Learning and Teaching of School Algebra. In D.A. Grows (Ed.), Handbook of Research on Mathematics Teaching and Learning (pp. 390–419). Macmillan, New York.
- . 2004. Algebraic Thinking in the Early Grades: What is It? In *The Mathematics Educator*. Vol. 8, No. 1. pp. 139–151.
- Koedinger, K.R. and A. Terao. 2002. A Cognitive Task Analysis of Using Pictures to Support Pre-algebraic Reasoning. In Proceedings of the Annual Meeting of the Cognitive Science Society 24 (24).
- LIAN, L.H. AND W.T. YEW. 2012. Assessing Algebraic Solving Ability: A Theoretical Framework. *International Education Studies*. Vol. 5, No. 6. pp. 177–188.
- LODHOLZ, R.D. 1990. The Transition from Arithmetic to Algebra. *Algebra for Everyone*. pp. 24–33.
- PILLAY, H., L. WILSS AND G. BOULTON-LEWIS. 1998. Sequential Development of Algebra Knowledge: A Cognitive Analysis. *Mathematics Education Research Journal*. Vol. 10, No. 2. pp. 87–102.
- SUBRAMANIAM, K. AND R. BANERJEE. 2011. The Arithmetic Algebra Connection: A Historical Pedagogical Perspective. In Jinfa Cai and Eric Knuth (Eds.), *Early Algebraization* (pp. 87–107). Springer, New York.
- SUGIARTI, L. AND H. RETNAWATI. 2019. Analysis of Student Difficulties on Algebra Problem Solving in Junior High School. In *Journal of Physics: Conference Series*, IOP Publishing. Vol. 1320
- USISKIN, Z. 1988. Conceptions of School Algebra and Uses of Variable. In A.F. Coxford and A.P. Shulte (Eds.), *The Ideas of Algebra, K-12 1988 Yearbook of the National Council of Teachers of Mathematics* (pp. 8–19). NCTM, Reston, VA.
- WARREN, E. 2003. The Role of Arithmetic Structure in the Transition from Arithmetic to Algebra. *Mathematics Education Research Journal*. Vol. 15, No. 2. pp. 122–137.