Richa Goswami
goswami84@gmail.com

Framework of Teacher's Knowledge: Concerns for Teacher Preparation Courses

Abstract

Teachers are the most important factor in the educational outcomes and learning levels achieved by any child, class or school. The paper presents a framework for teacher's knowledge and is based on the analysis of a small study assessing the knowledge of the teacher's from a few schools of Udaipur, Rajasthan. The findings of the study raise some pertinent recommendations and areas of concerns to be included while planning pre-service and in-service teacher capacity building programmes.

Teacher knowledge is characterised as a multidimensional construct, consisting of a variety of interacting components, such as general pedagogical heuristics, content specific pedagogical strategies and knowledge of the domain itself.

(Fennema & Frank, 1992)

In the case of mathematics, many previous studies (Ma, 1999 and Dewan & kumar , 2005) have reported about teachers' inability at solving mathematical questions and also that those who are able to solve the question are often unable to explain their solutions. For teachers, it is important to be able to articulate the process and thinking in solving a particular problem. These articulations provide students with an opportunity to become aware of problem solving strategies which are different from the algorithm of a particular kind of question. This forms an important part of the pedagogic role of the teacher. A simple question in this regard is the difference between multiplication and division. In the standard algorithm of multiplication, the digits are multiplied from the right whereas in division it happens from left. Teachers who can confidently apply the algorithm and get the answer are often not able to discuss why there is a difference in the algorithm. (Dewan & Kumar, 2005)

The connection between content and pedagogic knowledge is in a way quiet obvious as one cannot give suitable examples without knowing the subject well. But, there are other aspects of pedagogic knowledge like belief in the learning potential of the child or how children learn, which are independent of the subject knowledge.

It is organised in three sections. In the first section the important terms have been operationalised. The second section presents the understanding of teachers knowledge that I started with as a practitioner and the third section presents the findings from the study.

An Attempt Towards Defining Teacher's Knowledge

Teacher's knowledge can be seen as having there broad components: Content knowledge, Cognitive abilities and pedagogic knowledge.These are further sub-divided in Figure 1

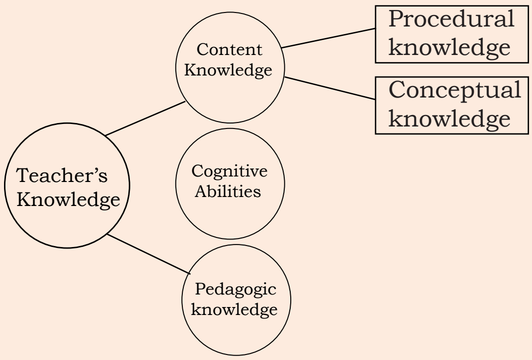

Figure 1. Framework of Teacher's Knowledge

There is a close connection between content knowledge and pedagogic knowledge as the latter is very much dependent on how well one understands the subject that is to be taught, our awareness of the structure of the subject, its underlying principles, its cognitive demands etc.

To make a valid comment about the knowledge package of the teachers, it is important to analyse how they understand and explain a particular topic. We shall now operationalize the most important and frequently used terms in this study, i.e. Content Knowledge (Procedural and Conceptual Knowledge), Cognitive abilities and Pedagogic Knowledge.

Component 1

Content Knowledge: Content knowledge refers to the knowledge of the subject to be taught. Content knowledge available with an individual can be broadly divided into two categories: Procedural and Conceptual knowledge(Ma, 1999 & IGNOU, 2000). Ma used the phrase "knowledge package" to represent how the mathematical knowledge is stored in a teacher's mind. She points out that the elements contained in a knowledge package are usually the same in both cases whether teachers have procedural or conceptual knowledge with difference in the way they are organized and the relationships that the teacher is able to make explicit in the process of explanation. The difference also lies in how conscious the teachers are of the elements and organization of the elements in their knowledge package.

(a) Procedural Knowledge: As the term suggests, procedural knowledge involves knowledge of the procedures or standard process of doing something. An example of this is the knowledge of algorithm to add 2 two digit numbers. Knowing that we need to start from the units column, carry over if the total is more than 10 and so on is being in possession of the procedural knowledge of addition of two-digit numbers.

This includes knowledge and explanation of the procedures or algorithms of doing a mathematical problem. The important point that Ma emphasized is that the process of teaching calls for some kind of explanation being given by the teacher. The two points that help us conclude that whether this explanation is an indicator of procedural knowledge or not are:

- Explanations may not really be mathematical or mathematically correct

  Liping Ma gives the example of U.S. teachers explanation of regrouping while subtracting 9 from 21, where the subtrahends ones digit number is bigger than the minuends ones digit number.

  "You can't subtract a bigger number from a smaller number... You must borrow from the next column because the next column has more in it. (Ms. Fay)

  But if you do not have enough ones, you go over to your friend here who has plenty. (Tr. Brady)

  "We can't subtract a bigger number from a smaller one" is a false mathematical statement. Although second graders are not learning how to subtract a bigger number from a smaller number, it does not mean that in mathematical operations one cannot subtract a bigger number from a smaller number. In fact, young students will learn how to subtract a bigger number from a smaller number in the future. Although this advanced skill is not taught in second grade, a student's future learning should not be confused by emphasizing a misconception.

To treat the two digits of the minuend as two friends, or two neighbors living next door to one another, is mathematically misleading in another way. It suggests that the two digits of the minuend are two independent numbers rather than two parts of one number (Ma, 1999).

- Making connections is an important process of both teaching and learning. In the case of only procedural knowledge based explanations, the connections are to procedural aspects and not to the basic principles of mathematics.

(b) Conceptual Knowledge: This on the other hand, implies a conscious packaging of the elements in the knowledge package and entails connections between different topics. A deeper understanding of any subject implies both extensive and well connected knowledge base. This complex interconnection implies that different knowledge items do not stand alone and the possessor of these knowledge items is aware of the connections.

The teachers who are conceptually more assured of their mathematical understanding are aware of the connections between different concepts, how a particular concept develops and thus are able to make the connections between different topics explicit to their students, able to solve and make new problems, and able to see the reason behind errors and alternative frameworks of their students. In their teaching, they do not easily resort to mechanical memory based explanations but rely on mathematical arguments. Another important aspect of conceptual understanding is the knowledge of basic principles. E.g., in the understanding of number systems: the important basic principle involved is the rate of composing a higher value unit.

In the case of teaching, it is important to be able to make these connections explicit. It will also have implications on the pedagogic approach of the teachers. A teacher who believes the above mentioned point will give students opportunities to present their own understanding of a question and different ways of solving it.

Component 2:

Cognitive Abilities: Cognitive abilities include abilities needed to construct knowledge in a subject. These can further be divided into general i.e., those needed to learn any subject and mathematical, i.e., those that are specific to mathematical learning. The examples of general cognitive abilities includes classification, observation, following logical argument, abstraction, pattern recognition, generalisation etc. Examples of mathematical cognitive abilities include estimation, spatial and quantitative visualisation etc. Needless to say all the general cognitive abilitites are needed in the learning of mathematics as well.

Component 3:

Pedagogic Knowledge: The knowledge or understanding about how to teach is not an area in itself but is affected by content knowledge, perception about why a particular subject should be taught, perceptions about learner and learning process and social biases. It includes the attitude of teacher towards the subject to be learnt as well as the learner. Views on learning process include how individuals learn, how mathematical concepts are formed, what strategies or material support it and how to plan the process, etc.

There is also need to understand human development and child development to have faith in the intrinsic ability of a child to construct one's own knowledge. This also is important to have faith that all children can learn mathematics. Pedagogic knowledge also includes knowledge about instructional material, about ways of assessment, ability to come up with usefulexamples and representations, ability to differentiate between what work needs support and supervision and what can be done independently, ability to anslyse the source of error (Ball et.al., 2005)

Understanding the Terms:

To understand the content and pedagogical knowledge, let's take an example of number system. Number system involves understanding numbers; knowing how to write and represent numbers in different ways; recognizing the quantity represented by numbers, discovering how a number relates to another number or group of numbers and different number sets and their properties.

Content Knowledge

In the primary and intermediate grades, number sense includes skills such as counting; understanding place value in the context of base 10 number system; writing and recognizing numbers in different forms such as expanded, word, and standard; and expressing a number in different ways—5 is "4 + 1" as well as "7 - 2," and 100 is 10 tens as well as 1 hundred. Number system also includes the ability to compare and order numbers—whole numbers, fractions, decimals, and integers—and the ability to identify a whole number by an attribute—such as odd or even, prime or composite-or as a multiple or factor of another number or classify numbers into natural number, rational number, etc. with knowledge about the defining property of each. It thus, includes the relationship between different sets i.e. how the set of natural numbers is subsumed within the set of rational numbers or the set of real numbers subsumes all the others.

Pedagogical Knowledge

Pedagogical knowledge for teaching number systems includes knowing that to learn counting, children initially need concrete materials but to really develop the concept of numbers they need to move away from the concrete. And this is true for all other devices as well. The aids are like crutches to be used if and when needed and not essential part of the mathematics classrooms. It includes knowing that to master a concept children need to engage with it in different contexts and need opportunity to use it in natural situations. It includes the awareness of how abstract the notion of numbers and number sets is.

What do we know about Teachers' understanding?

In the earlier section, we have discussed how the study has defined the phrase teacher's knowledge. In this section, an attempt has been made to articulate what we understand about teacher's knowledge based on our interactions with them in classrooms and workshops.

From 2009 to 2014, Vidya Bhawan Education Resource Centre (VBERC)[1] was engaged in working with government and Low Fee charging Private (LFP) schools of the city. The team was also engaged in conducting workshops for teachers and providing them support in the classrooms. We also participated in the Sarva Shiksha Abhiyaan workshops. This section is based on the reflections on these interactions

Content Knowledge

Procedural Knowledge: Most of

1 VBERC is a voluntary organization working with many state governments. As part of the team I had the opportunity to work with teachers and teacher educators of several states and intensely interact with government and private school teachers in Udaipur, Rajasthan.

the teachers who had been teaching mathematics in school were fairly comfortable with the procedural knowledge aspect of the subject, implying that they could solve the questions given in the textbook. If a new textbook was introduced or a new class was added to their timetable, they could in a year's time learn how to solve the questions in that book. But, this did not lead to the ability to explain why a particular method worked to provide the answer which in other words meant that they knew the algorithm but do not always knew the reason behind it. As a result even during workshops when there was discussion around the conceptual part of a topic, they usually started by using some phrases related to the topic but were unable to articulate a comprehensive picture. Also, in a setting where discussions about the underlying principles were made explicit, they were both amazed and interested and also felt that if this was shared with their students, it would help them in understanding and would probably lead to better retention. In spite of teaching a topic for many years, it was rarely seen that the teachers abstracted something from it or dug the reason behind it and therefore they in turn did not expect their students to be able to learn anything (more than taught) on their own.

Conceptual Knowledge: One can also confidently say that teachers were aware of the nature of mathematics as a hierarchical subject as they repeatedly said that to understand (or learn) a topic in mathematics, it was important that students knew the prior concepts. They also seemed to understand that the topics in the textbook were arranged in a hierarchical manner and the later chapters built on the initial chapters. As a result they did not skip the chapters and usually follow the sequence of chapters.

Cognitive Abilities: Based on the experience of attending the workshops as a facilitator, I can say that almost half of the teachers showed a grasp of many of the cognitive abilities in a general sense. But they rarely seem to apply them while working with mathematical problems. As soon as confronted with mathematical problems whether from the textbook or otherwise, there is an over reliance on known algorithms.

Let us take the example of knowledge and learning of quadrilaterals. Different kinds of quadrilaterals are introduced to students in different classes and also with the definition, similarities and variations between other quadrilaterals. Most teachers are able to identify a particular quadrilateral or on being told the specifications place it into a particular type of quadrilateral. This is a demonstration of procedural knowledge. But they can also categorise the quadrilaterals into mutually exclusive groups. Conceptual knowledge would be demonstrated in being able to see the relationships between them and realise that a square is a special type of rectangle or rhombus or parallelogram etc. and hence is also in those categories, this is recognised by only about half of the teachers.

Pedagogic Knowledge: Learning in Mathematics like other subjects was considered a matter of memorising and practicing the same or similar question again and again. Teaching was to aid this process of memorising and practicing. Teachers' perspective of mathematics learning is knowing the correct methods and solving the questions. As a result classroom pedagogy is mainly about solving the questions given in the exercises i.e., the classroom interaction in mathematics class is dominated by the question of how and not why. The procedural knowledge of solving questions can be seen at three levels:

- Ability to solve a particular question
- Ability to solve similar questions when a cue to identify the type is known. E.g. When you see “kul” in a queston you are supposed to add.
- Ability to solve questions of a particular topic.

The pedagogy is heavily dependent on the textbook and moves from exercise to exercise. The pages in between the exercises (especially in the NCERT textbooks) try and introduce the concept, help children understand the origin/ logical explanation of an algorithm or formulae. But the mathematics pedagogy of the teachers makes no allowance for reading the chapters in the class. Reading is not considered a skill important for mathematical learning; this may explain why children are unable to solve word problems. Also, the classrooms are dominated by the teacher and her monologue and rarely have legitimacy for students speaking anything. The dominant teacher speech is an indication of the understanding of learning process which focuses on the teacher telling and students listening. The learner is perceived as a passive recipient of the knowledge being given by the teacher. They do not see the possibility or worthiness of children exploring their surrounding and constructing their own mathematical knowledge. As discussed earlier, the teachers do seem to understand that mathematical topics are linked to each other and are in a hierarchy; but, at the same time the classroom transaction is very much linear. Most classes have students who do not know many topics from the previous classes, but the teaching plans of teachers do not ever go back to build on the base concepts and then move forward. Another important thing is the ability to make new questions. Teachers in schools are never seen as designing new questions (to pose realistic problems, stimulate thinking etc.) for the students, the only visible practice is of changing the numbers in the already given questions in the textbooks. The kind of pedagogic practice we see in most of the classes is very easily replaceable by a guidebook and that is exactly what we see happening. If a child misses some classes, she is expected to complete it by copying either from a friend’s notebook or from the guidebook. In some cases, even the most regular children are seen copying from the guidebook and as a result they are much ahead from what the teacher in the class is doing.

Another important aspect of pedagogical knowledge is the understanding of a teacher and learner’s role in the classroom and how do children learn. Teachers saw a very limited role for both. Teachers were limited to transacting what was given in the books. In the case of mathematics this implies presenting to the students the correct way of solving the questions. Learners were seen as those who would develop the ability to imitate the process and then be able to transfer this process/algorithm to similar questions. I call this ‘limiting’ as these roles trap both the learner and the teacher in roles which inhibit creativity, make the process of learning and teaching boring and ultimately alienates both the learner and the teacher from the wonders of the subject.

Understanding of a learner’s role includes how much and what to expect from them. There are studies (Rosenthal and Jacobson, 1968) showing that the teacher’s expectation of what students are capable of learning impacts their potential to learn immensely. Teachers that we were interacting with had very fixed ideas about mathematics learning and their students. They seemed to hold a fastidious belief that not all children can learn mathematics. They also felt that mathematics is a difficult subject and thus can be mastered only by intelligent children. This tells us that teachers also categorise their class into intelligent, average and below average students. Teachers often also feel that a subject like mathematics is difficult for children coming from disadvantaged backgrounds. As a result, the pedagogy of mathematics teachers seems to be very limiting and not expecting much from their students; thus, the possibility of the students abstracting their own patterns, forming rules, tackling new mathematical problems understanding on their own, logically proving something is very low.

A study to explore teacher's knowledge was initiated with dual motivation. On one hand it would provide empirical basis to assess the and ground the reflective understanding of the practitioners, like myself. And on the other hand a detailed analysis would provide direction to plan interventions in school and further work with teachers. The findings of the study have been presented in the following section.

About the Study

VBERC undertook a study to sketch a picture of teachers' knowledge in language and mathematics. This paper is based on the mathematics section of the study. The study sample included 49 teachers of whom 41 were women and 8 were men. The other important variable in the data was the management pattern of the school in which the teachers work: 18 teachers were from private schools and 31 teachers were from government schools. All the teachers were given a background questionnaire and a subject paper. Among those who scored high in the paper eight teachers were selected, four each engaged in the teaching of mathematics and language.

All the teachers were required to undertake Mathematics and Hindi competency test which contained various open-ended questions on nature & abilities of subject and related concepts. They were also asked to fill up questionnaire, gathering information on background of the teachers and their responses to pedagogical issues, process of learning, views about learner & learning material.

The mathematics paper explored three types of knowledge. The mathematics question paper used for the study included 16 questions. The questions were taken from the upper primary mathematics textbooks. 50% of the mathematics paper was based on conceptual; 24% on procedural and 26 % on cognitive abilities. Areas explored in the paper were number sense, geometry, data handling, fractions, mensuration, probability and unitary method.

Findings

The performance in the mathematics paper presented a very disappointing picture where the average percentage score was 39.7%. The mean score in questions assessing the conceptual knowledge was 23% and that of procedural knowledge was 47%. Both the scores are abysmally low. 59 % teachers scored 50 % or more in the procedural questions whereas only 11% teachers scored 50 % in conceptual questions.

Examples of questions

Procedural	पाँच मजदूर एक काम को 40 दिन में करते हैं तो 8 मजदूर उस काम को कितने दिन में करेंगे?
Conceptual	अभाज्य संख्या के विषय में 3 सही वाक्य छाँटकर लिखे। (a) 0 अभाज्य संख्या नहीं है। (b) 1 सबसे छोटी अभाज्य संख्या है। (c) 17×11 से बनी संख्या अभाज्य संख्या होगी। (d) 2 सबसे छोटी अभाज्य संख्या है। (e) प्रत्येक प्राकृत संख्या को 1 व अभाज्य संख्याओं के गुणनखण्डों के रूप में लिखा जा सकता।

The following question was not considered as part of any category but was placed in either of the two based on the response:

संख्या 5378 में कितनी दहाइयाँ हैं? नीचे दिए गये सही विकल्पों पर सही का निशान लगाइए। ध्यान रखिए एक से ज्यादा विकल्प भी सही हो सकते है।

(a) 7 दहाईयाँ (b) 37 इकाईयाँ

(c) 53 दहाईयाँ (d) 537 इकाईयाँ

If the teacher selected three options (a, b and d) then the answer was considered as evidence of conceptual knowledge and when only one option was chosen (a) then the answer was considered as evidence of procedural knowledge.

Table 1*

Mean score Math	Mean score in procedural knowledge related questions	Mean score in conceptual knowledge related questions
39.7	47	23.4

(* the scores are in percentages)

The background data collected from the teachers informed us that the Govt. school teachers were more experienced and had more opportunity to attend workshops. This reflected in their performance in questions related to attitude (classroom process and pedagogy) and mathematics as compare to private schools (18 private school and 31 Govt. school).There was a significant correlation seen in the experience and teacher training workshops attended by teachers and their score in attitude related questions, questions assessing number sense and the overall mathematics score.

The other significant correlation was seen in the performance of teachers in topics like algebra and data handling with more years of education. Higher the education a teacher has had, better was her performance in the above mentioned areas.

Similarly, teachers who gave positive responses to reading habit related questions also performed better in attitude related questions. Reading habit was assessed through questions about reading newspaper, magazines and books. Attitude scale included 24 questions exploring the views about learning, multilinguality, nature of mathematics and nature of language, classroom processes, multigrade teaching, new textbooks and teaching-learning material. Significant correlation implies that those teachers who reported that they read more gave more positive or progressive responses on the above questions.

Similarly, high correlation was seen in content knowledge of mathematics and pedagogical knowledge needed for teaching mathematics. As discussed earlier content knowledge is essential for part of the pedagogic knowledge. But as seen above the correlation with general pedagogy was also seen. Thus showing an overall correlation between general pedagogy and content knowledge of mathematics.

To conclude, one can say that both the kinds of teacher's knowledge were correlated amongst each other and were directly correlated with education level of the teacher, opportunity to attend workshops and reading habit.

What does it say about Teacher Education?

The above analysis and reflections on workshops shows us that content knowledge is an important aspect of teachers' knowledge. On one hand it forms the basis of what is taught to the children but as discussed above it also affects the pedagogic knowledge and implies that it decides how the subject is taught. How a teacher views a particular subject decides how she will teach it. When a teacher is confident of her content knowledge she is able to set new questions and is not restricted to the ones given in the textbook. She is also confident of solving any question thus would allow children to ask questions. In contrast a teacher who can only solve questions of a type would not allow students to freely ask questions. Similarly a confident teacher with conceptual clarity would be able to analyse the errors and also guess where a child is going by seeing her work and listening to her queries.

And yet, most teacher preparation programs assume the content knowledge, i.e. the students coming to the course already possess it as a result of their school and graduate education. The Bachelor of Elementary Education Program, in Delhi University makes an attempt at breaking away from th assumption and introduces papers like core mathematics, core natural science and core social science. The other example of integrated program dealing with both content and pedagogic knowledge is the Certificate program in teaching of primary school mathematics. The course provides reading material in easy language discussing how children learn and also important areas of elementary mathematics. There is a need for other teacher education program to be reviewed in this light and more discipline related content to be added to the course.

The second important learning from the research is regarding the reading habits. The teacher education programs need to ensure that the one or two years that the learners spend with them provides them ample opportunity to explore good quality literature and helps fire the reading interest. This requires a well stocked-curated library. There is also need to introduce original writings instead of always providing "notes" or "course material". At the same time assessment reforms that focus on assessing sustained learning and capture the change in thinking of the trainees are needed. Perhaps projects, practicums and detailed term papers would provide opportunity to assess the learning of the students better than the end of term examination.

The NCERT textbooks make an attempt at interacting with both the learner and the educator. The note to the teacher in the initial pages of the textbook discusses the expected way of using the book."There should be space for children to discuss ideas amongst themselves and make presentations as a group regarding what they have understood from the textbooks and present examples from the contexts of their own experiences. They should be encouraged to read the book in groups and formulate and express what they understand from it"(NCERT, 2006). The note emphasizes both the need to read text, discuss ideas and relate mathematics learning to real life situations, which are beyond the book. The life experiences and contexts of both the learners and the teachers are much richer than the contexts presented in the book. The note also emphasizes the need to encourage group work and learning from peers. Teachers can structure interesting tasks and problems to be worked out collectively and in small groups.

The third recommendation is about inservice teacher education programs. One sees the need for ongoing refresher programs or workshops to be conducted both for government school teachers and private school teachers. The design and content in these workshops need to be planned with utmost care. The teachers often complain about the facilities at the training centres, the preparation and knowledge of the resource person and the content of the workshop. Another aspect of inservice education programs should be to encourage and facilitate peer learning.

There is a need to ensure good education to all children irrespective of whether they study in government schools or private schools. Thus there is need for state inservice teacher education programs to cover the teachers from both government and private schools.

References

- Fennema, E., & Frank, M.L. (1992). Teachers' Knowledge and Its Impact. Handbook of Research on Mathematics Teaching and Learning. USA: National Council of Teacher of Mathematics.
- Shulman, L. (1987). Knowledge and Teaching: Foundations of the New Reform. Harvard Educational Review: April 1987, Vol. 57, No. 1, pp. 1-23.
- Ma, L. (1999). Knowing and teaching elementary mathematics teachers' understanding of fundamental mathematics in China and the United States. New York. Routledge.
- Ball, D.L., Hill, H. C, & Bass, H. (2005). Knowing mathematics for teaching: Who knows mathematics well enough to teach third grade, and how can we decide? American Educator, 30(3), p. 14–17, 20–22, 43–46.
- Rosenthal, R. and Jacobson, L. (1968). Pygmalion in the classroom: Teacher expectation and pupil's intellectual development. New York: Holt, Rinehart and Winston.
- Indira Gandhi National Open University. (2000). Certification in teaching of Primary Mathematics. New Delhi.
- Dewan, H.K. and Kumar, A.(2005). About Learning Mathemtics. In Agnihotri, R. and Bagchi, T. (Ed). Proceedings of Seminar on Construction of Knowledge. Udaipur. Vidya Bhawan Education Resource Centre.
- National Council of Educational Research and Training. (2006). Note to the Teacher. Mathematics: Textbook for Class VI. New Delhi