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Building a Better Mathematics Classroom

Abstract

This Paper focuses on the holistic teaching-learning process for Mathematics as embedded in the current thinking on Mathematics education which argues for the child building her own understanding through the process of engagement with concepts in different ways including having opportunities to collectively construct definitions and examine and modify them, developing formulas or/and algorithms, proofs and solving new problems, building patterns and generalisation, creating and setting problems and puzzles, talking and expressing mathematical concepts in written form, etc.

1. Introduction

Whenever we think of the methodology of teaching a subject some basic aspects have to be kept in mind. These include:

A. The nature of the subject and the way it manifests itself and constructs and accepts new knowledge;

B. The feelings and attitude of the learners, teachers, the system and the community to the subject and how they view its importance and its learnability;

C. The understanding about how humans, particularly children, learn;

D. The background of children and teachers along with the kind of schools and the education system that exists;

E. The purpose for which we want to educate children, in this case in Mathematics;

F. The curriculum and the content of Mathematics for the classes children are in and the resources that can be used. The class-room depends on the way these are understood. It is not possible to discuss all of them in this but we would discuss the essential aspects of most of them. Each of these have been discussed briefly to lay down the contours and their implications on the class-rooms.

2. Nature of the Subject and its Manifestation

The nature of the subject and the way it manifests itself and constructs and accepts new knowledge informs the way classrooms must be constructed. Mathematics is considered different from other disciplines; less driven by context, more abstract and in comparison to other disciplines more hierarchical implying each idea is intricately linked to other mathematical ideas and objects. This would suggest a linear class-room content and process. However, the recent discourse on Mathematics education elaborates the above and this brings out nuances that qualify above statements. While these nuances are stated in many different ways, we choose to follow one that is also reflected in the position paper on Mathematics NCERT 2005. It points out:

– Mathematics learning is about knowing, finding and creating patterns, relationships and discovering other properties among mathematical objects.

– Mathematics is present in what we do and is best learnt through that.

– Knowing Mathematics means being able to create and use ideas to form new ideas for yourself. It is about leading learners to find solutions to unfamiliar problems and to create new problems.

All these add to what Mathematics is and what is worth learning and teaching in it. These would be embedded in the chosen content areas like numbers, shapes, angles, data, functions, equations and their relationships to each other and to life, etc. This decade old formulation is far from being operational in the classrooms. The common notion of Mathematics continues to be limited to jugglery with numbers and different combinations of operations on them. The other areas are added as definitions and algorithms (standard and non-standard short-cuts). The attempt is to acquire enough bits and pieces to go through the exam without it all coming together to form a picture. In the absence of recognition of even the need to differentiate a mathematical statement from a non-mathematical statement or to understand what it means to generalise and to prove them. The students do not get a feel for the process of accepting mathematical knowledge as valid.

The other difficulty is that the NCF like statements are nuanced and has ideas about Mathematics that seem to suggest opposite ways. One argument that can be made is that Mathematics is fully a part of our life and has meaning only when completely embedded in that and hence as a utilitarian area of knowledge. There is a consequent over-focus on concrete materials and models. The above are however nuanced and have to be read together with other statements of the NCF. While Mathematics does relate to our lives and is always manifest in it, yet the form that it is seen in daily life has to be generalized and the underlying Mathematics abstracted from the experiences. It must be delinked from any particular context and from physical materials that may have been used as models for ideas. Validity of mathematical ideas is not established through multiple repeated observations to conclude the veracity of the statement. They have to be logically deduced from axioms and prior known results. Mathematics is about generalising rules and being able to see and understand them and its ideas emerge from precise presentation of the underlying concepts and their consistent use to build a hierarchically structured understanding.

3. Attitudes to Mathematics

"The feelings and attitude of the learners, teachers, the system and the community to the subject and how they view its importance and its learn ability also affects classrooms."_The present day perceptions of Mathematics as it is to be taught and learnt are characterized by some fundamental underlying commonalities:

- Mathematics is difficult and meant for a few intelligent children.
- Mathematics is not about understanding concepts but abilities to do.
- Mathematics is not useful to learn as it does not aid our day to day life except in the utilitarian sense of calculations and data.
- Mathematical knowledge and all other knowledge is learnt bit by bit through the process of simplification and explanation. These are what the child must reproduce exactly in the way they are given to her.
- Mathematics definitions, shortcuts and solutions to problems are to be explained and given to children and that they would then memorise and remember them.
- Repeated practice of the same algorithm on the same kind of problems can lead to conceptual understanding.

- In total therefore, Mathematics is a subject in which algorithms, facts and in fact even solutions to problems have to be remembered and reproduced.

These perceptions have led to a classroom methodology that believes that practice makes a learner perfect. An often quoted couplet about the constant rubbing of a rope against a stone for a long time resulting in a mark on the stone is used as an illustration of the maxim. We need to examine what this view means, especially with reference to our ideas and understanding of how children learn. It also leads to strategies that are stratified as most children considered not destined for Mathematics be only given short cuts to memorize and attention be given to those who are learning and hence also appear keener.

The above described commonalities run counter to not just the nature and purpose of Mathematics, but also principles of pedagogy, articulated in the NCFs, particularly in NCF 2005. NCF 2005's view, aligned with learnings from across the world, argues for conceptual and procedural understanding arrived through solving making new problems, alongwith formulation of own definitions and arguments. All this has ramifications for the role of the teacher, the teaching-learning process and the nature of resources .

The knowledge of children prior to school: A robust system to help all students learn Mathematics is helped by recognition of the extent to which Mathematics penetrates our lives. Any approximate or exact task of quantification, requires enumeration, estimation, comparison, scaling, use of the operations, conceptual ability to deal with space and spatial relations including transformations, visualising, mapping and projecting and so on. It is not only adults who learn and use all these but also children.

These are not however, learnt or even used in the formal way as in school and later Mathematics. Mathematics learnt and used in everyday life is often not useable in all situations and contexts that formal Mathematics considers similar. For example a newspaper vendor can track the papers sold and money collected; a vegetable seller or a paan seller can do all accounts needed for maintaining the shop, but would perhaps find it difficult to do the same type of mathematical processes when placed in another shop, even with the price list.

It is not that the Mathematics learnt in daily life is not at all generalised and abstracted. The situations presented are varied. The forms of generalisation, the way of reaching answers, the way of communicating, etc are all however, different from formal Mathematics. The important point is that some Mathematics is known to all children and each child not only acquires that, but develops her own strategies to deal with the Mathematics that she needs.

In this sense Mathematics has an important role in our life and our experience of it helps us learn more of it too. This also tells us that human children have the ability to learn to generalise, abstract, visualise and deal with quantities.

4. Learning Formal Mathematics

The above discussion suggests that Mathematics when acquired in a generalised and somewhat formal way would develop abstract thinking, logical reasoning and imagination. It would enrich life and provide new dimensions to thinking. The struggle to learn abstract principles would develop the power to formulate and understand arguments and the capacity to see interrelations among concepts. This enriched understanding would also help us deal with abstract ideas in other subjects and in our lives. Supporting us to understand and make better patterns, maps, appreciate area, volume and similarities between shapes and implications of their sizes. As Mathematics includes many aspects of our life and our environment, the symbiosis, inter dependence and inter-relationship between learning and using Mathematics in life needs to be emphasised.

We also need to ensure that even though Mathematics deals with a lot of symbols, abstractions, logic, spatial perceptions, generalisations, patterns and rules, it must not appear as difficult and meaningless to children. A classroom must give children a feeling that they are doing something meaningful that relates to reality around particularly at the primary school stage without getting trapped in utilitarianism. We must embed Mathematics and problems in it, for children to do through meaningful enjoyable situations and space for childrens' creativity and allow for multiple strategies thereby valuing children's articulation and logical formulations, even though not fully aligned. The classroom process must allow and demand that children create tasks, questions and problems for the classroom discussion. The NCF 2005 would suggest that the way forward is towards Mathematics class-rooms where children have opportunities to explore mathematical ideas and models not necessarily nor primarily concrete.

The purpose of Mathematics learning thus becomes developing the ability to explore mathematical entities and add to what is known. A growing ability extending beyond the classroom to help the learner mathematise her experiences. Using concepts to perceive the world differently and widening the manner of organising and analysing experiences. Helping learners find mathematical ideas and find instances of where mathematical ideas may be useful and what they can tell us. We would like them to be able to create new ways to solve problems and develop the ability to find solutions to new problems. In order to be able to recognise situations where mathematical ideas may be useful, we must be able to convert them into mathematical descriptions and present them as mathematical expressions or statements that can be solved and interpreted.

The background of children and teachers, kind of schools and the education system that exist for learning, and choice of materials and tasks: While the principle of universal education and equity in the nature and quality of education demands that all learn Mathematics and the evidence of engaging with Mathematics in daily life shows that all can learn it.

The current common belief and attitude about is that Mathematics can be understood only by those few who are bright and 'intellectually' well endowed. Only they can understand and for the rest therefore it has to be memorisation of solutions, or short cuts or algorithms and formulas. NCF 2005 recognises the abstract nature of mathematical ideas and the distance some children may have from some of these, but considers these as constraints to be overcome through concern and respect for the child, her knowledge, language and culture. In all pre-NCF 2005 documents (except perhaps the NCF 2000, where it was not so clear) one common feature was the impression that Mathematics is not easy to learn and that many children will fail to learn it. The kind of ideas about the resources and the methods to help children learn were also different.

5. The curriculum and the content of Mathematics: While there is an agreement that Mathematics must be learnt by all, but what to teach in Mathematics remains contentious. The emphasis in Nai Talim and in the Kothari Commission was different as being core in one and useful for Science and engineering in the other. National policy on education 1986 focussed on this and ability to use Mathematics in daily life. The NCF 2000 underlined the importance of the utilitarian purposes for life of Mathematics. With the scope largely around numbers and their use in market, in mensuration or other areas in life.

The NCF 2005 with an emphasis on abstraction, use of logical forms, grasping, discovering, creating as well as appreciating patterns and new ideas brought in new focus on mathematisation giving a dialectic relationship to Mathematics to its daily use and opened space to discover Mathematics. Focus also shifted to developing concepts and new algorithms and learners' own ways of solving problems.

Since the NCF 2005, there is talk without clarification of making the Mathematics class-rooms constructivist. Constructivism in classrooms cannot mean that children rediscover all knowledge or form curriculum. The school Mathematics has to be suitable and useful for the stage of the children. Constructivism can shape the manner and pace of transaction within overall goals and expectations. Besides classrooms are not about individual children but about collective teacher assisted learning. Constructivism here means for each child space to think, formulate ideas, descriptions and definitions. A conceptual structure for the child that is open to change with challenges and new situations.

Constructivism does require that Mathematics classrooms consider learners as naturally exploratory, keen to learn and act. They need tasks to stretch their minds and challenge logical abilities, through discussions, planning, strategizing and implementing them. The Mathematics classroom should not desire blind application of not understood algorithm or one way of solving a problem but suggest many alternative algorithms and expect learners to also find new ones. Problems with scope for many different correct solutions must be included to develop nuanced understanding of concepts. Class room must involve all children and give space to do things at their own pace and in their own ways. Besides, children need opportunities to solve problems, reflect on solutions and examine the logical arguments provided to evaluate them and find loopholes. Learning Mathematics is not about remembering solutions or methods but knowing how to solve problems. Problem solving provides opportunities to think rationally, understand and create methods motivates students to become active participants and not passive receivers. This can help learners abstract, generalise, formulate and prove statements based on logic. In learning to abstract children would also need some concrete materials, experiences and known contexts scaffold to help them. In Mathematics we need to separate verification from proof and explanation from exploration and recognising that.

Mathematics not only helps in day-to-day situations, but also develops logical reasoning, abstract thinking and imagination. Enriched understanding developed through it helps us deal with abstract ideas in other subjects as well. These two dimensions have implications for the resources to be used and the way of using them.

6. Ideas on Resources in the Mathematics Classroom

The idea of materials in Mathematics classrooms has become fashionable. While, the nature and structure of classrooms, remains unchanged, terms such as 'engaging materials' and activities are widely used. The need to interrogate the nature of materials is more for Mathematics as it is not empirical or experimental. Empirical proofs like showing by measurements that the sum of the angles of a few triangle is 180 degree do not constitute a proof. Mathematics being abstract expects us to separate objects of Mathematics from their context. For example, numbers are independent of objects and the unit-ten-hundred system and operations on it not limited to bundles and sticks, place holder cards or any other models.

Resources for Mathematics teaching must align with the purpose of Mathematics teaching, nature of Mathematics, assumptions about the learner and teaching learning process. Printed materials and other concrete objects are examples of materials but, their use is different in Mathematics.

The other important point is that almost everything around us can be used as a concrete material to support learning of Mathematics. Stones, flowers, leaves, water, etc can help quantification. Some objects can aid in visualisation when placed in different positions at different angles, ways and then trying to anticipate how they would appear when changed. The materials presenting models as scaffold, help build foundations of mathematical ideas. Obviously, the use of these has to be linked to the textbooks, worksheets, etc.

It also must be recognised that the nature, specific purpose and use of the materials evolve as we move to different stages of learning Mathematics. This movement may not be entirely linear, but as we go towards building of mathematical ideas the nature of what may be constituted as a concrete experience changes and slowly the requirement and limitation of using concrete experiences to describe mathematical ideas starts becoming evident. Materials and concrete contexts are pegs to create temporary models and to help visualise and manipulate mathematical objects in the form of their concrete models till learners can do without them and deal with mathematical concepts without mediation.

As Mathematics develops hierarchically the earlier learnt abstract ideas become concrete models for further abstraction and formalisation. Base 10 system becomes the concrete model for generalised representation system in any base. Numbers and operations on them generalise to algebra. Geometrical ideas generalise from objects to nets to representations as faces and edges on two dimensional surface and their co-ordinates through point, line, plane, etc. The 3 D system, physically visualisable, moves to a n-dimensional system that has to be mentally visualised. We now look at some aspects of Mathematics and resources for it and different levels.

a) Teaching Mathematics in primary classes

We know that conversations around matematical concepts with opportunity for the learner to express and get feedback is useful for development of thought and conceptual structures. In addition concrete experiences and memories can form the basis of generalization and abstraction.

These are useful whenever new ideas are introduced but, are critical for the primary stage. They change according to the concept and the maturity of the learners and in primary classes would be informed by the struggle of children to read and use the textbook. One point in that is the language may differ from the common languages of learners, but there are difficulties with the way arguments are formulated as well. This implies an important role for language to help children learn concepts and more Mathematics.

In the absence of prior, familiar, easily usable and available abstract conceptual structures with the child, concrete models are critical for the primary classes. The recognition of Mathematics as a discipline emerging from some basic axioms and assumptions based on logical procedures requires concrete experiences and learners engaging with classification, matching, counting, etc. For this they may use their bodies and parts, stones, leaves, any other object in the classroom, games, scores, etc. as means. Dice with a Ludo or a snake and ladder board gives many such experiential opportunities. Pictures can be used for various comparing and matching tasks, identify groups that have more objects than the others. Each of these examples has a different degree of concreteness. However, for Mathematics 5 chairs represent 5 as well as 5 stones can. Five, is just the name of an idea relating to certain other ideas in a specific way. Concrete representations need to use different models for the varied situations and nuances of the concepts.

Promotion of thinking, exploring answers, their comprehension and analysis would not expect or follow from a blind application of un understood algorithm and should encourage children to find many different ways to solve problems. It must point out that many alternative adapted working algorithms and strategies exist and problems can have many different correct solutions and their analysis. The classroom must involve all children and give them space.

The obvious question is, can this be possible? The elements that are required have been mentioned above. To put theses together, the teacher needs to have scope for choosing what she needs to do and is capable of and then ask how she would proceed to fulfil above suggestions. Some suggestions as examples are: Divide grade 2 children into groups of 6-8 with one or two dice and a pile of stones for each group. Each child throws the dice and picks up stones on each turn to see who has more after 5 turns. They slowly abstract that two numbers together produce a third and relate to these and other numbers as abstract entities.

Similarly fractional numbers children can have many relevant examples from daily experience using unequal and equal parts of a whole. They can list and consider these. For example, including concrete models where the fractional number is more than one whole (e.g. 3/2) and recognising that the nature of whole is important.

Experiences of spatial relations, transformations, symmetry, congruence, patterns, measurements through opportunities to play with shapes, etc. build foundations for further development of ideas. The tasks for this can include building shapes, arranging them, observing and anticipating transformed forms, using themselves and surroundings as data sources for organisation and presentation of simple data using for example counters. In all this we know some children start enjoying the play with abstract entities sooner than others. Also different children enjoy play with different abstractions though all eventually began to use logic and understand Mathematics.

b) Teaching Mathematics in Upper Primary classes

Upper primary Mathematics is linked to experience, but moves further towards abstraction. Children yet need context and/or models linked to their experience to find meaning but eventually must work just with ideas. This challenge is to engage each child through context and move her from this dependence. So while the child should be able to identify principles useable in a context, she should not be limited to contexts. At this stage they may be asked to build their own models and use them as supports.

This stage links the more concrete and direct experience linked with Mathematics in primary classes to formal, less experience dependent abstract secondary Mathematics. This stage must acknowledge that many people after school would take different occupations, most not requiring formulae and algorithms. What all do require is mathematisation of understanding, a deep understanding and appreciation of Mathematics to sharpen analysis and maintenance of logical thread in thought.

Ideas like negative numbers, generalised fractional numbers, rational numbers, letter numbers, ideas like point, line, etc. introduced and developed in upper primary are not easy to model. The representations and examples used to introduce these can be confusing and are inaccurate unless dealt with care and with changing examples with including warnings about limitation and inherent inappropriateness of models.

Through upper primary to secondary classes, these models have to be discarded giving way to the characteristics of Mathematics. The concepts no longer be imbued with materials and tasks that are also different. The empirical and measurement aspect giving way to logic and proof. Concept building now would require more dialogue and at most consciously temporary modeling. The problem formulation and solving must expand here. Not remembering solutions or methods, but knowing how to solve problems, thinking rationally to create methods as well as processes. This motivates and makes active participants to construct knowledge rather than being passive receivers. Problem solving requires students to select or design possible solutions and revise or redesign the steps, if required. These are thus essential parts of the Mathematics classroom program.

7. Mathematics Lab and Beyond

a) In talking about resources and materials there is a lot of talk about Mathematics laboratory. It is important to better understand the purpose of materials in learning Mathematics and the notion of lab so that we use the possibilities in an appropriate manner. We know learning requires experiences related to the concepts being learnt, but Mathematics deals with ideas that are eventually with abstract ideas. For example, numbers are not related to the objects that are used to represent them, a function not related to the curve that depicts it, a triangle that has points of zero dimension and lines of zero thickness can only be visualized in the mind, etc.

With this and the recognition that Mathematics relationships are not empirically provable or verifiable means the the purpose and scope of Mathematics labs need to be sharpened. To illustrate, no amount of measuring angles of quadrilaterals can convince anyone that the sums of 4 interior angles would always be 360 degree. No model constructed would be free of experimental blemish to show exactly 360 degree. But this does not lead us to conclude that such figures do not have interior angles with sum equal to 360. So what is the purpose of materials in Mathematics learning? This example and question is true for all such uses of Mathematics lab including the oft used and quoted verification/demonstration of Pythagorous theorem

(b) This does not mean that there is no use for materials in Mathematics. They are for many purposes and stages not just useful, but essential in helping learners deal with abstract ideas initially and concretely visualise them. Materials in Mathematics learning while initially helping children experience the abstract ideas concretely have to be withdrawn eventually making the child constructed them in the mind and move away from concrete examples. For example starting from an angle or a ring as a model of a circular shape to a circle drawn on paper, we go through different stages of concreteness in the depiction of the idea of a circle, which can be only imagined as a shape bounded by a line of zero width. When we draw a chord and find the angle subtended at the centre we are dealing with representations of lines and angles. Representing a general circle by a diagram is crucial to understanding of the proof of statements about chord and other properties of a circle. In these we are not taking circles with specific lengths of radius and length of chords, but the generalized abstraction relation. In primary classes we encourage the use of a lot of concrete materials and this usage must drastically reduce through upper primary to secondary classes.

Learners may use these representations, but not see them as being mathematical objects themselves. Therefore, in the secondary classes, Mathematics lab can only provide opportunity to help children concretely visualise some of the ideas to which they have not been exposed earlier. Over emphasis on materials and expecting their use to prove statements can be extremely misleading and become a barrier for an appropriate understanding of Mathematics. The tasks in the so called "Mathematics activity room" therefore must help children explore ideas and start dealing with them in more abstract forms.

It has been pointed out above that the drawing of any geometrical shape is a model and a representation. A circle is not the line as drawn, but the locus of points equidistant from a point in a two dimensional plane. The plot of a function itself, seems far from a concrete reality, but is actually a representation of the more abstract relationship. The plot displays how the function behaves and shows its form.

(c) One example of a resource with significant possibilities is Geogebra. This is different from the usual as it allows creative exploration of graphs and curves as models of functions and use geometrical diagrams to represent and explore relationships. For example marking equal sides and angles, seeing symmetry, transformations and congruence, even constructions of shapes, etc. can be explored through Geogebra. It does not demonstrate but allows the user to model what she wants to explore. Its effective use becomes possible only when the user recognises the abstract nature of Mathematics and uses modelling through Geogebra recognise patterns and reach generalisations. Another example is of using coins, dice or coloured balls etc. to set up an experimental distribution of outcomes helps build understanding of chance, independence of events, probability, etc.

The Mathematics lab needs tasks that make learners explore ideas. The word itself denotes exploration and curious thinking not fixed and correct explanations. Lab therefore has to be multi-dimensional allowing learners to explore ideas and to add to their library of experiences. The focus of the lab must be aligned to objectives of Mathematics teaching, not trapped in explanation and telling syndrome with the recognition that material are temporarily scaffolded to form ideas. All this as a part of the classroom process and not a separate visit to an exotic location called mathematical lab or something like that.

The idea of the Mathematics lab, therefore, has to be in conjunction with the nature of mathematical ideas and questions of what materials, till when, for what and how they should be used need to be considered.

(d) Text Books as a resource in Mathematics

For organised transaction of knowledge along a certain syllabus, including content, abilities and perhaps dispositions as well, textbooks are a necessary evil. The text books in Mathematics are reputed to be dreary and unattractive as they are full of numbers, letter numbers, abstract geometrical shapes, unusual brackets and symbols interspersed by terse sentences. Their standard format suggests that learning mathematical ideas is about seeing examples and then following them to do similar things without conversation or dialogue. Context and experience is a post concept development application or a mere entry point.

The textbooks of Mathematics have to act at a major resource for the teachers to create engaging processes for learners. It has to indicate how to bring and use the experiences of the children, their mathematical ideas embedded in their own language, culture and their daily activities, and make the classroom inclusive, participative, exploratory with simultaneous focus on conceptualisation, formulation and articulation of ideas.

Secondly for the textbooks to be used in the spirit intended appropriate guidance, support and enabling ambience for the teachers and the learners has to be available. It should be able to struggle with the notion that Mathematics classroom and text book are needed for examples of solved problems with methods, techniques, short cuts and memory devices with guidance on how to use and replicate them. The textbook would be organised assuming that since Mathematics is hierarchically organised, learning would be organised similarly and once a topic has been covered, it can be revised by doing problems similar to the ones done earlier.

In the alternative perspective of Mathematics textbook must help the learner engage with mathematical ideas in different ways and experience the nuances. The materials should help elaborate and interlink her concepts embedding them in her language and experience. Hierarchical nature requires spiralling not linear sequence for concepts to become internalised. Learners must come back to the ideas explored on multiple occasions in different contexts and in alignment with different concepts.

The following note to the teacher from the textbook of the NCERT exemplifies not just the way the book is intended to be used, but also the way it has been organised;

"We have tried to link chapters with each other and to use the concepts learnt in the initial chapters to the ideas in the subsequent chapters. We hope that you will use this as an opportunity to revise these concepts in a spiraling way so that children are helped to appreciate the entire conceptual structure of

Mathematics. Please give more time to ideas of negative number, fractions, variables and other ideas that are new for children. Many of these are the basis for further learning of Mathematics. For children to learn Mathematics, be confident in it and understand the foundational ideas, they need to develop their own framework of concepts. This would require a classroom where children are discussing ideas, looking for solutions of problems, setting new problems and finding not only their own ways of solving problems, but also their own definitions language they can use and understand. These definitions yet need to be as general and complete as the standard definition."

The indication is explicit on spiralling, on the need for greater time, multiple contexts and nuances for internalising mathematical ideas and the way the classroom conversation architecture should be organised as well as the expectations from what children would achieve in this process.

As Mathematics in school moves across grades its structure and organisation must be such that the same ideas do not occur concentrically.

The textbooks and other materials must require Mathematics teacher to think and reflect on her class-room experiences and not move mechanically. Such textbooks also call for reasonably long and well-structured orientation program. They should pool new ideas and develop new activities to supplement the textbook.

The language used and the nature and extent of illustrations in it help reduce terseness of the textbook, making it comprehendible for the learner. The flow of the book must aid the learner to pause, reflect and engage with it. It must expect the learner to articulate ideas, concepts, explanations, generalisations, definitions and attempt to prove or disprove mathematical statements. The nature of the Mathematics books have started changing recently particularly after the NCF 2005, both at NCERT and in some states however, many remain in the earlier framework.

Some of the books also have illustrations that show children engaged in doing Mathematics differently. They are shown using resources, chatting and discussing with each other, workout, exploring, imagining and visualising.

The way the textbook is to be used and the nature of the classroom comes out well from this excerpt form the NCERT book *"There are many situations provided in the book where children will be verifying principles or patterns and would also be trying to find out exceptions to these. So while on the one hand children would be expected to observe patterns and make generalisations, they would also be required to identify and find exceptions to the generalisations, extend patterns to new situations and check their validity. This is an essential part of the ideas of Mathematics learning and therefore, if you can find other places where such exercises can be created for students it would be useful."* The key points here being that children are expected to explore, think and work out the answers to the problems. It expects teacher and childrens to create exercises and suggests that the teachers should look for more places for problems that could be located, created or found.

(8) Summing Up:

Mathematics subject in secondary classes includes elaborating and consolidating the conceptual edifice, to make logical and organised arguments, precisely and concisely formulate ideas, to perceive rules and generalization and found mechanisms to prove them. Go beyond numbers to understanding abstract number systems, their properties and general rules about them and similarly in other areas.

Gradually a broad and tenuous agreement on the universal purpose and scope of concepts for Mathematics in secondary classes has been arrived at. Spelt out in the NCF 2005, it is however, yet to reach the classrooms. With multiple formulations of its implications and approaches for the materials and the classroom architecture and processes there is no consensus on strategy to be adopted. On deeper analysis some of the differences in strategy seem to emerge from basic purpose and perspective differences. The unfortunate linking of a child engaging meaningful program to a confused terminology of child-centered or constructivist program has led to a feeling that a Mathematics or a school program could be evolved based on what children want to do on any particular day with an overdose of materials and physical activity. In a pragmatic formulation of meaningful school Mathematics program constructivism would imply the child space to think in the classroom, formulate her ideas, her descriptions and definitions with an attitude and a conceptual structure that is open to changes when presented with new kinds of challenges and situations.

We know that for many children class X is a means to study further, but in the context of Indian education, the secondary classes are the final year of general education and after this students would go in to different roles. A complete general education requires a rounded up Mathematics understanding and capabilities (not mere skills) that are needed by all citizens. In line with the international thinking, the NCF 2005 has enlarged the scope of this with focus on mathematization to attempt enrichment of the scope of thought and visualisation. The secondary school Mathematics, therefore, on the one hand, needs to focus on the consolidation of the conceptual edifice initiated in the classes 6 to 8, but also take it forward to help child explore wider connection and deeper understanding. The logical formulation and the arguments included in each step along with the precision of presentation is of value to engage with the world in more forceful manner.

To summarise, a much larger number of students are now attempting Mathematics as a part of their secondary program as we push towards universalisation of secondary education. The purposes of teaching Mathematics, the pedagogy for it and the materials for secondary classes has somewhat evolved over the last decade or so, but there is a need to put all this in some framework and much more thinking and clarifying is needed. All this has also brought forth the need for context and resources in the secondary classrooms which were earlier devoid of these. The materials in the class-room would not only be an aid to scaffold introduction, but also of engagement with concepts. Like in the upper primary classes and in fact now much more than that, the students need to be asked to create contexts and resources and present them rather than being given materials to manipulate as would be likely in the primary and occasionally in the upper primary classrooms.

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